Iranian Journal of Mathematical Sciences and Informatics
Vol. 14, No. 2 (2019), pp 9-18
DOI: 10.7508/ijmsi.2019.02.002

Hereditarily Homogeneous Generalized Topological Spaces

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Abstract. In this paper we study hereditarily homogeneous generalized topological spaces. Various properties of hereditarily homogeneous generalized topological spaces are discussed. We prove that a generalized topological space is hereditarily homogeneous if and only if every transposition of X is a μ-homeomorphism on X.

Keywords: Generalized topology, Homogeneous, Hereditarily homogeneous GTS, Highly transitive permutation groups, Bihomogeneous.

2000 Mathematics Subject Classification: 54A05, 54D10.

1. Introduction

Generalized topology is a generalization of the concept of topology. Á. Császár has developed a theory of generalized topological spaces [2, 3, 5, 4, 6, 7, 8]. A subfamily μ of $P(X)$ is called a generalized topology (GT) if $\emptyset \in \mu$ and μ is closed under arbitrary union. The pair (X, μ) is called generalized topological space (GTS)[2]. A $GTS\,(X,\,\mu)$ is said to be strong if $X \in \mu$. The elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets [2]. Let A be a subset of a GTS, $(X,\,\mu)$. Then μ-closure of A is the intersection of all μ- closed sets containing A, which is denoted by $c_\mu(A)$ and μ-interior of A, $i_\mu(A)$ is the union of all μ-open sets contained in A [4, 6]. Clearly A is μ-closed if and only if $A = c_\mu(A)$. Also we have $x \in c_\mu(A)$ if and only if every μ-open set containing x intersects A. Let Y be a subset of

Received 24 October 2016; Accepted 11 August 2017

a $GTS\,(X,\mu)$. Then $\mu_Y = \{Y \cap G : G \in \mu\}$ is called the subspace GT on Y, and the pair (Y, μ_Y) is called the subspace GTS on Y.

Let (X, μ) and (Y, λ) be GTS's and $f : (X, \mu) \to (Y, \lambda)$ be a mapping. Then f is said to be (μ, λ) continuous if $f^{-1}(M) \in \mu$ for all $M \in \lambda$ and f is said to be (μ, λ)-open ((μ, λ)-closed) if $f(M) \in \lambda$ ($f(N)$ is λ-closed) for every $M \in \mu$ (μ-closed set N) [2]. Also f is called (μ, λ)- homeomorphism if f is bijective, (μ, λ)- continuous and f^{-1} is (λ, μ)-continuous. Two GTS's (X, μ) and (Y, λ) are said to be generalized homeomorphic if there is a (μ, λ)-homeomorphism $f : (X, \mu) \to (Y, \lambda)$ [14].

The symmetric group $S(X)$ on a nonempty set X is the group consisting of all permutations or bijections of X [10]. A permutation group is a subgroup of the symmetric group. A permutation group A on X is called transitive if for every pair of points $x, y \in X$, there exists $f \in A$ such that $f(x) = y$ [10].

The set of all μ-homeomorphisms on a GTS onto itself forms a group under composition of functions. This group is called the group of μ-homeomorphisms of (X,μ) and is denoted by $H(X,\mu)$. Clearly the group of all μ-homeomorpisms of (X,μ) is a subgroup of the symmetric group on X.

In [15] V. Kannan and P. T. Ramachandran investigated the hereditarily homogeneous topological spaces and provided various characterizations for the same. Here we study hereditarily homogeneity in strong generalized topological spaces.

Throughout this paper by a GTS, we will always mean a strong generalized topological space. For any set G, $|G|$ stands for the cardinality of G.

2. Hereditarily Homogeneous Generalized Topological Spaces

Definition 2.1. [14] A $GTS\,(X,\mu)$ is said to be homogeneous if for any two elements $x, y \in X$, there exists a μ-homeomorphism h of (X,μ) onto itself such that $h(x) = y$.

In other words a GTS (X, μ) is homogeneous if the group of μ-homeomorphisms on (X,μ) is a transitive permutation group on X.

A subspace of a homogeneous GTS need not be homogeneous. See the following example.

Example 2.2. Let $X = \{a,b,c,d\}$ and $\mu = \{\emptyset, \{a,b\}, \{c,d\}, \{b,c\}, \{a,d\}, \{a,b,c\}, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, X\}$. Here

$$H(X, \mu) = \{I, (a\,b\,c\,d), (a\,b)(c\,d), (a\,d\,c\,b), (a\,c), (b\,d), (a\,b)(c\,d), (a\,d)(b\,c)\}$$

where I denotes the identity permutation on X. Clearly $H(X, \mu)$ is a transitive permutation group on X. So (X, μ) is a homogeneous GTS.
Now consider $Y = \{a,b,d\}$. The subspace GT on Y,

$$\mu_Y = \{\emptyset, Y, \{b\}, \{d\}, \{b,d\}, \{a,b\}, \{a,d\}\}.$$

Then (Y, μ_Y) is not a homogeneous GTS since $\{a\}$ is not μ-open in Y.

Definition 2.3. A $GTS\,(X,\mu)$ is said to be hereditarily homogeneous if every subspace of (X,μ) is homogeneous.

EXAMPLE 2.4. Let $X = \mathbb{Z}$ be the set of integers and define a generalized topology on X as

$$\mu = \{G \subset \mathbb{Z} : (\mathbb{Z} \setminus G) \cap 2\mathbb{Z} \text{ is finite}\} \cup \{\emptyset\}.$$

Then $(X,\, \mu)$ is a hereditarily homogeneous GTS. In fact $(X,\, \mu)$ is a topology on X.

Remark 2.5. From the definition of hereditarily homogeneous GTS, it follows that a hereditarily homogeneous GTS is homogeneous. But the converse is not true as shown in the Example 2.2.

Now we prove a hereditarily homogeneous GTS is either $\mu-T_1$ or indiscrete. First we recall a $\mu - T_1$ generalized topological space.

Definition 2.6. [11, 17] A GTS, $(X,\, \mu)$ is called $\mu - T_1$ if for any pair of distinct points x and y of X, there exists a μ-open set U of X containing x but not y and a μ-open set V of X containing y but not x.

A GTS, (X,μ) is $\mu - T_1$ if and only if the singletons of X are μ-closed in X [11, 17].

Theorem 2.7. *A hereditarily homogeneous GTS is either $\mu - T_1$ or indiscrete topological space.*

Proof. Let (X,μ) be a hereditarily homogeneous GTS which is not $\mu - T_1$. Then there exist two distinct elements x and y of X such that every μ-open set containing x contains y also.

Claim I: Any μ-open set in X contains x if and only if it contains y.

Consider the subspace $Y = \{x,y\}$ of (X,μ). Since (X,μ) is hereditarily homogeneous, (Y,μ_Y) is a homogeneous GTS. It follows that (Y,μ_Y) is either a discrete topological space or an indiscrete space. If $\{x\}$ is μ-open in Y, then there exists a μ-open set G in X such that $\{x\} = Y \cap G$ and $y \notin G$. This is not possible. So the $GTS\,(Y,\mu_Y)$ is the indiscrete space. Hence the claim.

Claim II: There exist no proper nonempty μ-open sets in X.

If possible let G be any proper nonempty μ-open set in X. Let us consider the following two cases.

Case 1: $x \in G$, $y \in G$

Since G is a nonempty proper μ-open set, we choose an element $z \in X \setminus G$. Now consider the subspace $Y = \{x,y,z\}$ of X. Then $Y \cap G$ is μ_Y-open in Y. This implies that $\{z\}$ is μ_Y-closed in Y. Suppose that $\{y\}$ is μ_Y-closed. Then $\{x,z\}$ is μ_Y-open. This implies that there exists a μ-open set H such that $\{x,z\} = Y \cap H$. So H is a μ-open set containing x but not y. This is not possible by Claim I. Thus we get $\{z\}$ is μ_Y-closed and $\{y\}$ is not μ_Y-closed.

This is a contradiction to the fact that $(Y,\, \mu_Y)$ is a homogeneous GTS.
Case 2: $x \notin G$, $y \notin G$
In this case choose $z \in G$. Here also consider the subspace $Y = \{x, y, z\}$. Then we get $\{z\}$ is a μ_Y-open set in Y. But $\{x\}$ and $\{y\}$ are not μ_Y-open by Claim I. So (Y, μ_Y) is not homogeneous. This is also a contradiction.

Thus in both cases we arrive at a contradiction and consequently there exist no proper nonempty μ-open sets and hence $(X,\, \mu)$ is an indiscrete topological space. □

Definition 2.8. [12, 13] Let $(X,\, \mu)$ be a GTS and $G \subset X$. Then G is called μ-dense if $c_\mu(G) = X$.

Lemma 2.9. *Let (X, μ) be a hereditarily homogeneous GTS. Let x, y be two elements of X and G be a subset of X which contains neither x nor y. Then $x \in c_\mu(G)$ if and only if $y \in c_\mu(G)$*

Proof. Let $Y = G \cup \{x, y\}$. Then (Y, μ_Y) is a hereditarily homogeneous GTS. So there exists a homeomorphism $h : Y \to Y$ such that $h(x) = y$. Then

$$h(G) = \begin{cases} G & \text{if } h(y) = x \\ (G \setminus \{h(y)\}) \cup \{x\} & \text{if } h(y) \neq x. \end{cases}$$

Case 1: $h(G) = G$.
Then $x \in c_\mu(G)$

$$\begin{aligned} &\Longleftrightarrow h(x) \in h(c_\mu(G)) \\ &\Longleftrightarrow y \in c_\mu(h(G)) \\ &\Longleftrightarrow y \in c_\mu(G). \end{aligned}$$

Case 2: $h(G) = (G \setminus \{h(y)\}) \cup \{x\}$.
Let $x \in c_\mu(G)$. Then every μ-open set containing x intersects G. Suppose there exists a μ-open set U containing y such that $U \cap G = \emptyset$. Since $x \in c_\mu(G)$, we have $y = h(x) \in h(c_\mu(G))$. Since h is a μ-homeomorphism, we have $h(c_\mu(G) = c_\mu(h(G))$. This follows that every μ-open set containing y intersect $G \setminus h(y) \cup \{x\}$. So $U \cap (G \setminus h(y) \cup \{x\}) \neq \emptyset$. This implies that U is a μ-open set containing x and hence $U \cap G \neq \emptyset$. This is a contradiction. So every μ-open set containing y intersect G. Thus $x \in c_\mu(G)$. Similarly if $y \in c_\mu(G)$, using the fact that h^{-1} is μ-continuous, we get $x \in c_\mu(G)$.

This completes the proof. □

Theorem 2.10. *Let $(X,\, \mu)$ be a hereditarily homogeneous GTS. Then every subset of X is either μ-closed or μ-dense in X.*

Proof. If $(X,\, \mu)$ is an indiscrete topological space, there is nothing to prove. Now suppose that (X, μ) is a $\mu - T_1$ space. Let G be a subset of X which is not

μ-closed. Then by Lemma 2.9 it follows that $X \setminus G \subset c_\mu(G)$. So G is μ-dense in X. □

Corollary 2.11. *Let (X, μ) be a hereditarily homogeneous GTS.*

(1) *The μ-closure of each μ-open set is μ-open in X.*
(2) *If G and H are disjoint nonempty μ-open sets, then G and H are μ-closed in X.*

Proof. (1). Let G be a μ-open set in X. Then by Theorem 2.10, G is either μ-closed or μ-dense in X. If G is μ-closed, then $c_\mu(G) = G$, which is μ-open. Otherwise $c_\mu(G) = X$, which is also μ-open. This completes the proof.
(2). Since G and H are disjoint nonempty μ-open sets, $G \subset X \setminus H$ and $H \subset X \setminus G$. So

$$c_\mu(G) \subset c_\mu(X \setminus H) = X \setminus H \text{ and } c_\mu(H) \subset c_\mu(X \setminus G) = X \setminus G.$$

Consequently by Theorem 2.10, $c_\mu(G) = G$ and $c_\mu(H) = H$. □

Remark 2.12. In a hereditarily homogeneous GTS, a μ-dense subset need not be μ-open.

EXAMPLE 2.13. Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here (X, μ) is a hereditarily homogeneous GTS and $G = \{a, b\}$ is a μ-dense subset of X which is not μ-open.

Definition 2.14. [13] let (X, μ) be a GTS. Then (X, μ) is said to be submaximal if each μ-dense subset of (X, μ) is a μ-open set.

From Remark 2.12, it is clear that a hereditarily homogeneous GTS need not be a submaximal space.

EXAMPLE 2.15. Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, X, \{a, c\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here (X, μ) is a submaximal space but not a homogeneous GTS. So a submaximal GTS need not be a hereditarily homogeneous GTS.

Corollary 2.16. *Let (X, μ) be a hereditarily homogeneous and submaximal GTS. Then every subset of X is either μ-closed or μ-open in X.*

Proof. Proof is obvious from Theorem 2.10. □

Definition 2.17. [5] A $GTS\,(X, \mu)$ is extremely disconnected if the μ-closure of each μ-open set is μ-open.

Corollary 2.18. *Every hereditarily homogeneous GTS is extremely disconnected.*

Proof. Proof follows from Corollary 2.11. □

But an extremely disconnected GTS need not be hereditarily homogeneous.

Example 2.19. Let $X = \{a, b, c, d\}$ and

$$\mu = \{\emptyset, \{d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, a\}, X\}.$$

Then (X, μ) is an extremely disconnected GTS but not hereditarily homogeneous.

Theorem 2.20. *A GTS (X, μ) is hereditarily homogeneous if and only if every transposition of X is a μ-homeomorphism on X.*

Proof. Let (X, μ) be a hereditarily homogeneous GTS. Let a and b be two distinct elements of X and f be the transposition $(a\,b)$ mapping a to b, b to a and keeping all other elements fixed. To prove that f is a μ-homeomorphism of (X, μ) onto itself, it suffices to show that K is μ-closed in X if and only if $f(K)$ is μ-closed for every μ-closed subset K of X. Let K be a μ-closed set. Consider the following cases.

Case(1): $a \notin K$, $b \notin K$.
Then $f(K) = K$ and hence $f(K)$ is μ-closed in X.

Case(2): $a \in K$, $b \in K$.
In this case also $f(K) = K$ and hence $f(K)$ is μ-closed in X.

Case(3): $a \in K$, $b \notin K$.
Here $f(K) = (K \setminus \{a\}) \cup \{b\}$.
If $f(K)$ is μ-closed, there is nothing to prove. Suppose that $f(K)$ is not μ-closed in X. By Theorem 2.10, it follows that $f(K)$ is μ-dense in X.

Let $Y = K \cup \{b\}$. Since K is μ-closed in X, $K \cap Y = K$ is μ_Y-closed in Y. So $\{b\}$ is μ_Y-open in Y. Since the subspace Y is homogeneous, $\{b\}$ is μ_Y-open implies that all singletons are μ_Y-open in Y. In particular $\{a\}$ is μ_Y-open in Y. This implies that there exist a μ- open set U containing a in X such that $\{a\} = U \cap Y$. So

$$U \cap (K \setminus \{a\}) \cup \{b\} = U \cap (Y \setminus \{a\}) = \emptyset.$$

This is a contradiction to the fact that $(K \setminus \{a\}) \cup \{b\}$ is μ-dense in X.
Thus $f(K)$ is μ-closed in X.

Case(4): $a \notin K$, $b \in K$.
Similar to Case(3).

Now let $f(K)$ be a μ-closed set. Then by what we have proved, $f(f(K)) = K$ is a μ-closed set in X. So $f = (a\,b)$ is a μ-homeomorphism of (X, μ) onto itself. Thus every transposition of X is a μ-homeomorphism of X onto itself.

Conversely assume that every transposition of X is a μ-homeomorphism on X. It follows that (X, μ) is a homogeneous GTS. Let (Y, μ_Y) be a subspace of X and a, $b \in Y$. Then by our assumption, $f = (a, b)$ is a μ-homeomorphism on X. Let U be μ_Y-open in Y. Then $U = Y \cap G$ where G is μ-open in X. So

$$\begin{aligned} f(U) &= f(Y \cap G) \\ &= Y \cap f(G) \text{ since } f \text{ is a bijection.} \end{aligned}$$

Since f is a μ-homeomorphism, $f(G)$ is μ-open in X and hence $f(U)$ is μ_Y-open in Y. Since f is a transposition, we have $f^{-1} = f$. So f is a μ_Y-homeomorphism on Y which maps a to b. Thus (Y, μ_Y) is homogeneous. Since Y is arbitrary, X is a hereditarily homogeneous GTS. Hence the proof. □

Remark 2.21. For $n \leq |X|$, we say that a permutation group A is n-transitive if given two n-tuples $(x_1, x_2, \ldots x_n)$ and $(y_1, y_2, \ldots, y_n)$ of distinct elements, there exists $f \in A$ with $f(x_i) = y_i$ for $i = 1, 2, \ldots n$. When X is infinite, we say that A is highly transitive if it is n-transitive for all positive integers n [1]. Since every transposition is a μ-homeomorphism of a hereditarily homogeneous GTS, all finite permutations on X are also μ-homeomorphisms. Thus the group of all μ-homeomorphisms of an infinite hereditarily homogeneous GTS is a highly transitive permutation group.

Lemma 2.22. *Let (X, μ) be a GTS such that every transposition is a μ-homeomorphism on X onto itself. Then supersets of μ-open sets are μ-open in X.*

Proof. Let G be a μ-open set in (X, μ) and $G \subset H \subset X$. If $G = H$, there is nothing to prove. Otherwise choose two elements $a \in G$ and $b \in H \setminus G$. Let $f = (a, b)$. Then $f(G) = G \setminus \{a\} \cup \{b\}$ is a μ-open set since f is a μ homeomorphism on X. Now $G \cup f(G) = G \cup \{b\}$, which is a μ-open set. Thus we get $H = \underset{b \in H \setminus G}{\cup} (G \cup \{b\})$ is μ-open. This completes the proof. □

For any set X, a nonempty collection of subsets $\mathcal{S}$ of X is said to form a stack if $\emptyset \notin \mathcal{S}$ and $A \in \mathcal{S}$, $X \supset B \supset A$ implies $B \in \mathcal{S}$ [16].

Definition 2.23. A stack $\mathbb{S}$ is called free if $\underset{s \in \mathbb{S}}{\cap} S = \emptyset$.

Corollary 2.24. *Let (X, μ) be a $\mu - T_1$ hereditarily homogeneous GTS. then $\mu \setminus \{\emptyset\}$ is a free stack.*

Proof. Proof follows from Theorem 2.7 and Lemma 2.22. □

Remark 2.25. Let (X, μ) be a GTS such that $\mu \setminus \{\emptyset\}$ is a free stack on X. Then (X, μ) need not be hereditary homogeneous.

Example 2.26. Let $X = \{a, b, c, d\}$ and $\mu = \{\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\} \{a, c, d\}, X, \emptyset\}$. Here $\mu \setminus \{\emptyset\}$ is a free stack but (X, μ) is not a hereditarily homogeneous GTS.

Definition 2.27. [9] A $GTS\,(X, \mu)$ is said to be completely homogeneous if the group of μ-homeomorphisms on (X, μ) is the symmetric group on X.

In [9] P. M. Dhanya characterized completely homogeneous GTS.

Theorem 2.28. *The only completely homogeneous strong GTS on an arbitrary set X are the following* [9].

(1) $\{G \subset X : |G| \geq m\} \cup \{\emptyset\}$ *where m is a cardinal number and $m \leq |X|$.*
(2) $\{G \subset X : |X \setminus G| < m\} \cup \{\emptyset\}$ *where m is a cardinal number and $m \leq |X|$.*
(3) $\{G \subset X : |X \setminus G| \leq m\} \cup \{\emptyset\}$ *where m is a cardinal number and $m < |X|$.*

Proposition 2.29. *Every subspace of a completely homogeneous GTS is completely homogeneous.*

Proof. Let Y be a subset of a completely homogeneous GTS and $f \in S(Y)$. Let $U \in \mu_Y$. Then $U = Y \cap G$ where G is μ-open in X. Now $f(U) = f(Y \cap G) = Y \cap f(G)$ where $f(G)$ is μ-open in X. Thus f is a μ_Y-open map. Similarly we can prove f^{-1} is also μ_Y-open map. Thus f is a μ_Y-homeomorphism on X. This completes the proof. □

Proposition 2.30. *Every completely homogeneous GTS is hereditarily homogeneous.*

Proof. We have every completely homogeneous GTS is homogeneous and a subspace of a completely homogeneous GTS is completely homogeneous by the Proposition 2.29. So every completely homogeneous GTS is hereditary homogeneous. □

Remark 2.31. A hereditarily homogeneous GTS need not be completely homogeneous. In Example 2.4, every transposition is a μ-homeomorphism on X. So by Theorem 2.20, (X, μ) is a hereditarily homogeneous GTS. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(x) = x+1$, $x \in \mathbb{Z}$ and $G = 2\mathbb{Z}$, which is a μ-open set. Now $f(G) = 2\mathbb{Z}+1$, which is not a μ-open set. So f is not a μ-homeomorphism and hence (Z, μ) is not a completely homogeneous GTS.

Now we characterize finite hereditarily homogeneous GTS.

Proposition 2.32. *Let (X, μ) be a finite GTS. Then (X, μ) is completely homogeneous if and only if it is hereditarily homogeneous.*

Proof. From Proposition 2.30, we have that every completely homogeneous GTS is hereditarily homogeneous. Conversely let (X, μ) be a hereditarily homogeneous GTS. Then by Lemma 2.20, every transposition of X is a μ-homeomorphism of X onto itself. Since X is finite, every bijection of X can be expressed as composition of transpositions. Thus every bijection is a μ-homoemorphism on X. This completes the proof. □

Remark 2.33. The only hereditarily homogeneous GT on a finite set X is $\{G \subset X : |G| \geq n\} \cup \{\emptyset\}$ where $1 \leq n \leq |X|$.

Definition 2.34. A $GTS\,(X,\,\mu)$ is said to be bihomogeneous provided that for every two points $x,\,y \in X$ there is a μ-homeomorphism $f : X \to X$ such that $f(x) = y$ and $f(y) = x$.

A homogeneous GTS need not be bihomogeneous.

Proposition 2.35. *Every hereditarily homogeneous GTS is bihomogeneous.*

Proof. Let $(X,\,\mu)$ be a hereditarily homogeneous GTS. Then by Theorem 2.20 every transposition of X is a μ- homeomorphism. So every hereditary homogeneous GTS is bihomogeneous. □

A bihomogeneous space need not be hereditary homogeneous.

EXAMPLE 2.36. Let $X = \mathbb{R}$ and μ be a generalized topology on X having the base $\mathbb{B} = \{(-\infty,\,a),\,(b,\,\infty) : a, b \in \mathbb{R}\}$. Then $(X,\,\mu)$ is a bihomogeneous GTS but not hereditary homogeneous.

Acknowledgments

The valuable suggestions and guidance from my supervising guide Dr. Ramachandran. P. T. during the preparation of this paper is greatly acknowledged. The author is thankful to the referees and editors for their valuable comments and suggestions which improved this paper.

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