Iranian Journal of Mathematical Sciences and Informatics
Vol. 20, No. 1 (2025), pp 151-174
DOI: 10.61186/ijmsi.20.1.151

Some Characterizations of Γ−semihypergroups by Soft Generalized Γ-hyperideals

Sabahat Ali Khan[a], Mohammad Yahya Abbasi[a], Kostaq Hila[b*], Ahmad Raza[a]

[a]Department of Mathematics, Jamia Millia Islamia, New Delhi-110 025, India
[b]Department of Mathematical Engineering, Polytechnic University of Tirana, Albania

E-mail: khansabahat361@gmail.com
E-mail: mabbasi@jmi.ac.in
E-mail: kostaq_hila@yahoo.com
E-mail: arhanraza@gmail.com

Abstract. The aim of this paper is to establish a relationship between soft sets and Γ-semihypergroups. In this aspect, we have introduced soft intersection generalized interior Γ-hyperideals and soft intersection generalized bi-Γ-hyperideals of Γ-semihypergroups with some interesting examples. Moreover, we study some characterizations of regular, intra-regular, semisimple and right weakly regular Γ-semihypergroups in terms of soft intersection generalized interior Γ-hyperideals and soft intersection generalized bi-Γ-hyperideals.

Keywords: Γ-semihypergroups, Soft generalized Γ-hyperideals, Regular and intra-regular Γ-semihypergroups, Interior simple Γ-semihypergroups.

2010 Mathematics subject classification: 20N20, 03E75.

*Corresponding Author

Received 12 May 2020; Accepted 06 May 2021

1. Introduction

Marty [31] introduced the natural generalization of classical algebraic structures, known as algebraic hyperstructures. In a classical algebraic structure, the composition of two elements is an element while in algebraic hyperstructures, the composition of two elements is a non-empty set. Hasankhani [23] defined ideals in right(left) semihypergroups and introduced the hyper versions of Green's relations.

The notion of Γ-semigroup was introduced by Sen [36] as a generalization of semigroups and ternary semigroups. Several authors extended some results of semigroups to Γ-semigroups. Davvaz et al. [24] introduced the notion of Γ-semihypergroup as a generalization of Γ-semigroup. They defined Γ-hyperideals in Γ-semihypergroups. For other results and examples, refer to [1, 5, 15, 16, 25, 26, 39].

Sardar et al. [35] introduced and fuzzified the notion of interior ideal in Γ-semigroups. Feng and Corsini [20] defined interior ideal of an ordered Γ-semigroup. They introduced the notions of (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup. Ersoy et. al [17] introduced the notions of interior Γ-hyperideal and fuzzy interior Γ-hyperideal in Γ-semihypergroup and defined interior simple Γ-semihypergroup. Recently, Tang et. al [41] introduced interior Γ-hyperideals of ordered Γ-semihypergroups and discussed fuzzy interior Γ-hyperideals of ordered Γ-semihypergroups.

Soft set theory is basically the generalization of fuzzy set theory which was introduced by Molodtsov [33] to deal uncertain and complex problems in many areas such as engineering, medical science, environmental science, social science etc. because this types of problems cannot be dealt by classical methods. Maji et al. [30] presented some basic algebraic operations on soft sets and provided an analytical approach to the theory of soft sets. Researches work on soft set theory and its applications in various fields are progressing rabidly. The algebraic structures of soft sets have been studied by Feng et al. [19, 21] on semirings and semigroups, Aktas and Cagman [9, 3] on groups, Ma and Zhan [29], and Zhan et al. [44] on hemirings and so on. Anvariyeh et. al. [6] initiated soft semihypergroups by using the soft set theory. Naz and Shabir [34] investigated the basic terms and properties of soft sets. They relate soft sets with the concept of semihypergroups. Farooq et al. [18] characterized regular and left regular ordered semihypergroups using intersection soft generalized bi-hyperideals (see also [38]). In [2], ordered Γ-semihypergroups in terms of soft intersection Γ-hyperideals were studied. Feng et. al. [22] studied on several types of soft subsets and various soft equal relations and in [21], they introduced soft binary relations. In [28], Liu et. al. generalized soft M-subsets and soft M-equal relations in a natural way. Khademan et. al. [27] introduced fuzzy soft positive implicative hyper BCK-ideals and investigated their several properties.

Recently, several papers have been published on soft set theory and hyperstructure theory. However, studies on the fundamental structure of Γ-semihypergroups in terms of soft set theory remained untouched. Therefore, this finding encouraged the need to make a such study and generalize some known results on Γ-semihypergroups using soft set theory. The main motive of this paper is to study some structural properties of Γ-semihypergroups applying the soft set theory. In this paper, the concepts of soft generalized interior Γ-hyperideals and soft generalized bi-Γ-hyperideals of Γ-semihypergroups are introduced and some characterizations of regular, intra-regular, semisimple and right weakly regular Γ-semihypergroups in terms of soft generalized Γ-hyperideals are studied.

Throughout the paper we represent:
$\mathbf{S}$: Γ-Semihypergroup
$\mathcal{V}$: an initial universe
E : a set of parameters
$F(\mathbf{S})$: set of all soft sets of $\mathbf{S}$ over $\mathcal{V}$
$P(\mathcal{V})$: the powerset of $\mathcal{V}$ and A, B, $C \subseteq E$.

2. Preliminaries

2.1. Γ-semihypergroup.

Definition 2.1. [12, 14] Let $\mathbf{S}$ be a non-empty set and let $\wp^*(\mathbf{S})$ be the set of all non-empty subsets of $\mathbf{S}$. A hyperoperation on $\mathbf{S}$ is a map $\circ : \mathbf{S} \times \mathbf{S} \to \wp^*(\mathbf{S})$ and $(\mathbf{S}, \circ)$ is called a hypergroupoid.

Definition 2.2. [12, 14] A hypergroupoid $(\mathbf{S}, \circ)$ is called a semihypergroup if for all x, y, z of $\mathbf{S}$ we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$$

If $x \in \mathbf{S}$ and A and B are non-empty subsets of $\mathbf{S}$, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \; x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

Definition 2.3. [24] Let $\mathbf{S}$ and Γ be two non-empty sets. $\mathbf{S}$ is called a Γ-semihypergroup if every $\gamma \in \Gamma$ is a hyperoperation on $\mathbf{S}$, i.e, $x\gamma y \subseteq S$ for every x, $y \in \mathbf{S}$, such that, for $a, b, c, d \in \mathbf{S}, \gamma, \gamma_1 \in \Gamma, a = c, b = d, \gamma = \gamma_1$ imply $a\gamma b = c\gamma_1 d$ and for every α, $\beta \in \Gamma$ and x, y, $z \in \mathbf{S}$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$, which means that $\bigcup_{u \in x\alpha y} u \; \beta \; z = \bigcup_{v \in y\beta z} x \; \alpha \; v$.

If every $\gamma \in \Gamma$ is an operation, then $\mathbf{S}$ is a Γ-semigroup.

If $(\mathbf{S}, \gamma)$ is a hypergroup for every $\gamma \in \Gamma$, then $\mathbf{S}$ is called a Γ-hypergroup.

Let A and B be two non-empty subsets of $\mathbf{S}$ and $\gamma \in \Gamma$. We define:

$$A\gamma B = \cup\{a\gamma b \mid a \in A,\, b \in B\}. \text{ Also}$$

$$A\Gamma B = \cup\{\, a\gamma b \mid a \in A,\, b \in B \text{ and } \gamma \in \Gamma \,\} = \bigcup_{\gamma\,\in\,\Gamma} A\gamma B.$$

Definition 2.4. [24] A non-empty subset T of a Γ-semihypergroup $\mathbf{S}$ is called sub-Γ-semihypergroup of $\mathbf{S}$ if $t_1\ \gamma\ t_2 \subseteq T$ for every $t_1,\, t_2 \in T$ and $\gamma \in \Gamma$.

Definition 2.5. [24] A non-empty subset I of a Γ-semihypergroup $\mathbf{S}$ is called a left(right) Γ-hyperideal of $\mathbf{S}$ if $\mathbf{S}\ \Gamma\ I \subseteq I(I\ \Gamma\ \mathbf{S} \subseteq I$, respectively) and is a Γ-hyperideal of $\mathbf{S}$ if it is both a left and a right Γ-hyperideal.

Definition 2.6. [24] Let A be a non-empty subset of a Γ-semihypergroup $\mathbf{S}$. Then, intersection of all hyperideals of $\mathbf{S}$ containing A is an ideal of $\mathbf{S}$ generated by A, and denoted by $< A >$.

Lemma 2.7. [24] *Let $\mathbf{S}$ be a Γ-semihypergroup. If A is a non-empty subset of $\mathbf{S}$, then*

$$< A > = A \cup A\Gamma\mathbf{S} \cup \mathbf{S}\Gamma A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S}.$$

Similarly, $< A >_r = A \cup A\Gamma\mathbf{S}$ and $< A >_l = A \cup \mathbf{S}\Gamma A$. If $A = \{a\}$, then we write $< \{a\} >_r = < a >_r$ and $< \{a\} >_l = < a >_l$.

Definition 2.8. [4] A sub-Γ-semihypergroup B of $\mathbf{S}$ is called a bi-Γ-hyperideal of $\mathbf{S}$, if $B\Gamma\mathbf{S}\Gamma B \subseteq B$.

Definition 2.9. [17] A sub-Γ-semihypergroup I of $\mathbf{S}$ is called an interior-Γ-hyperideal of $\mathbf{S}$, if $\mathbf{S}\Gamma I\Gamma\mathbf{S} \subseteq I$.

Definition 2.10. [17] A Γ-semihypergroup $\mathbf{S}$ is called an interior simple Γ-semihypergroup, if it has no proper interior Γ-hyperideal.

Lemma 2.11. [17] *Let $\mathbf{S}$ be a Γ-semihypergroup $\mathbf{S}$. For any $a \in \mathbf{S}$, $\mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S}$ is an interior Γ-hyperideal of $\mathbf{S}$.*

Lemma 2.12. [17] *Let $\mathbf{S}$ be a Γ-semihypergroup. Then $\mathbf{S}$ is an interior simple if and only if $\mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S} = \mathbf{S}$ for all $a \in \mathbf{S}$.*

Definition 2.13. [43] A Γ-semihypergroup $\mathbf{S}$ is called regular, if for each $s \in \mathbf{S}$ there exists $x \in \mathbf{S}$ such that $s \in s\ \Gamma\ x\ \Gamma\ s$.

Definition 2.14. [43] A Γ-semihypergroup $\mathbf{S}$ is called intra-regular, if for each $a \in \mathbf{S}$ there exist $x,\, y \in \mathbf{S}$ such that $a \in x\ \Gamma\ a\ \Gamma\ a\ \Gamma\ y$.

2.2. **Soft Sets.**

Definition 2.15. [8, 33] A soft set $\mathcal{F}_A$ over $\mathcal{V}$ is a set defined by $\mathcal{F}_A : E \to P(\mathcal{V})$ such that $\mathcal{F}_A(x) = \emptyset$ if $x \notin A$.

Here $\mathcal{F}_A$ is also called an approximate function. A soft set over $\mathcal{V}$ can be represented by the set of ordered pairs

$$\mathcal{F}_A = \{(x,\ \mathcal{F}_A(x))\ :\ x \in E,\ \mathcal{F}_A(x) \in P(\mathcal{V})\}.$$

It is clear to see that a soft set is a parameterized family of subsets of the set $\mathcal{V}$.

Definition 2.16. [8] Let $\mathcal{F}_A$, $\mathcal{F}_B \in F(\mathbf{S})$. Then, $\mathcal{F}_A$ is called a soft subset of $\mathcal{F}_B$ and denoted by $\mathcal{F}_A \sqsubseteq \mathcal{F}_B$, if $\mathcal{F}_A(x) \subseteq \mathcal{F}_B(x)$ for all $x \in E$.

Definition 2.17. [8] Let $\mathcal{F}_A$, $\mathcal{F}_B \in F(\mathbf{S})$. Then, union of $\mathcal{F}_A$ and $\mathcal{F}_B$ denoted by $\mathcal{F}_A \widetilde{\bigcup} \mathcal{F}_B$, is defined as $\mathcal{F}_A \widetilde{\bigcup} \mathcal{F}_B = \mathcal{F}_{A\tilde{\cup}B}$, where $\mathcal{F}_{A\tilde{\cup}B}(x) = \mathcal{F}_A(x) \bigcup \mathcal{F}_B(x)$ for all $x \in E$.

Definition 2.18. [8] Let $\mathcal{F}_A$, $\mathcal{F}_B \in F(\mathbf{S})$. Then, intersection of $\mathcal{F}_A$ and $\mathcal{F}_B$ denoted by $\mathcal{F}_A \widetilde{\bigcap} \mathcal{F}_B$, is defined as $\mathcal{F}_A \widetilde{\bigcap} \mathcal{F}_B = \mathcal{F}_{A\tilde{\cap}B}$, where $\mathcal{F}_{A\tilde{\cap}B}(x) = \mathcal{F}_A(x) \bigcap \mathcal{F}_B(x)$ for all $x \in E$.

Definition 2.19. Let Y be a subset of $\mathbf{S}$. We denote the soft characteristic function of Y by $\mathcal{S}_Y$ and is defined as:

$$\mathcal{S}_Y(y) = \begin{cases} \mathcal{V},\ if\ y \in Y \\ \emptyset\ if\ y \notin Y. \end{cases}$$

Definition 2.20. Let $\mathcal{F}_A$ be a soft set over $\mathcal{V}$ and $\delta \subseteq \mathcal{V}$ Then, upper δ-inclusion of $\mathcal{F}_A$ is denoted by $\mathcal{V}(\mathcal{F}_A\ ;\ \delta)$ and is defined as

$$\mathcal{V}(\mathcal{F}_A\ ;\ \delta) = \{x \in A \mid \mathcal{F}_A(x)\ \supseteq\ \delta\}.$$

In this paper, we denote a Γ-semihypergroup $\mathbf{S}$ as a set of parameters.

Let $\mathbf{S}$ be a Γ-semihypergroup. For $x \in \mathbf{S}$, we define $\$_x = \{(y,z) \in \mathbf{S} \times \mathbf{S} \mid x\ \in\ y\ \Gamma\ z\}$.

Definition 2.21. Let $\mathcal{F}_S$ and $\mathcal{G}_S$ be two soft sets of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$. Then soft product $\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S$ is a soft set of $\mathbf{S}$ over $\mathcal{V}$, defined by

$$(\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S)(x) = \begin{cases} \bigcup\limits_{(y,z)\in\$_x} \{\mathcal{F}_S(y) \cap \mathcal{G}_S(z)\} \text{ if } \$_\mathrm{x} \neq \emptyset \\ \emptyset \text{ if } \$_\mathrm{x} = \emptyset \end{cases}$$

for all $x \in \mathbf{S}$.

Theorem 2.22. *Let X and Y be non-empty subsets of a Γ-semihypergroup $\mathbf{S}$. Then*
(1) If $X \subseteq Y$, then $\mathcal{S}_X \sqsubseteq \mathcal{S}_Y$
(2) $\mathcal{S}_X \widetilde{\bigcap} \mathcal{S}_Y = \mathcal{S}_{X\cap Y}$, $\mathcal{S}_X \widetilde{\bigcup} \mathcal{S}_Y = \mathcal{S}_{X\cup Y}$.
(3) $\mathcal{S}_X \,\hat{\diamond}\, \mathcal{S}_Y = \mathcal{S}_{X\Gamma Y}$

Theorem 2.23. *Let $\mathbf{S}$ be a Γ-semihypergroup and $F(\mathbf{S})$ be the set of all soft sets of $\mathbf{S}$ over $\mathcal{V}$. Then, for any $\mathcal{F}_S$, $\mathcal{G}_S$ and $\mathcal{H}_S \in F(\mathbf{S})$,*

$$((\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{H}_S) = (\mathcal{F}_S \,\hat{\diamond}\, (\mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S)).$$

Definition 2.24. A non-null soft set $\mathcal{F}_S$ is said to be an S.I. sub Γ-semihypergroup of **S** over $\mathcal{V}$ if

$$\bigcap_{\vartheta \in x\Gamma y} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(x) \cap \mathcal{F}_S(y) \;\forall\; x,\, y \in \mathbf{S}.$$

Definition 2.25. A non-null soft set $\mathcal{F}_S$ is said to be an S.I. left(resp., right) Γ-hyperideal of **S** over $\mathcal{V}$ if

$$\bigcap_{\vartheta \in x\Gamma y} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(y) \text{ (resp. } \bigcap_{\vartheta \in x\Gamma y} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(x)) \;\forall\; x,\, y \in \mathbf{S}.$$

Definition 2.26. A non-null soft set $\mathcal{F}_S$ is said to be an S.I. Γ-hyperideal of **S** over $\mathcal{V}$ if it is both an S.I. left and an S.I. right Γ-hyperideal of **S** over $\mathcal{V}$.

3. Soft generalized Γ-hyperideals

In this section, the notions of soft intersection generalized interior Γ-hyperideals (briefly, S.I. generalized interior Γ-hyperideals) and soft intersection generalized bi-Γ-hyperideals (briefly, S.I. generalized bi-Γ-hyperideals) of Γ-semihypergroups are introduced and some useful results with respect to the soft intersection product (briefly, S.I. product) are studied.

Definition 3.1. A non-empty subset I of a Γ-semihypergroup **S** is called a generalized interior-Γ-hyperideal of **S**, if $\mathbf{S}\Gamma I\Gamma\mathbf{S} \subseteq I$.

Definition 3.2. A non-empty subset B of a Γ-semihypergroup **S** is called a generalized bi-Γ-hyperideal of **S**, if $B\Gamma\mathbf{S}\Gamma B \subseteq B$.

Let $A(\neq \emptyset) \subseteq \mathbf{S}$ and let

$$\mathbf{I} = \{I \mid \text{ I is a generalized interior } \Gamma-\text{hyperideal of } \mathbf{S} \text{ contaning A }\}.$$

Then $\mathbf{I} \neq \emptyset$, because **S** itself is a generalized interior Γ-hyperideal.
Let $< A >_{i_{gen}} = \bigcap_{I \in \mathbf{I}} I$. Then $< A >_{i_{gen}}$ is a generalized interior Γ-hyperideal of **S**. $< A >_{i_{gen}}$ is a smallest generalized interior Γ-hyperideal of **S** containing A and $< A >_{i_{gen}}$ is called generalized interior-Γ-hyperideal of **S** generated by A. Analogously, we define smallest generalized bi-Γ-hyperideal of **S** containing A and we denote it by $< A >_{b_{gen}}$.

Theorem 3.3. *For a non-empty subset A of a Γ-semihypergroup **S**,*

$$< A >_{i_{gen}} = A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S}.$$

Proof. Let $I = A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S}$, then for every $a \in A$, $s_1, s_2 \in \mathbf{S}$ and $\gamma_1, \gamma_2 \in \Gamma$, we have

$$\begin{aligned} s_1\gamma_1 a\gamma_2 s_2 &\subseteq \mathbf{S}\Gamma I\Gamma\mathbf{S} \\ &= \mathbf{S}\Gamma(A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S})\Gamma\mathbf{S} \\ &= \mathbf{S}\Gamma A\Gamma\mathbf{S} \cup \mathbf{S}\Gamma\mathbf{S}\Gamma A\Gamma\mathbf{S}\Gamma\mathbf{S} \\ &\subseteq \mathbf{S}\Gamma A\Gamma\mathbf{S} \\ &\subseteq I. \end{aligned}$$

Therefore, I is a generalized interior-Γ-hyperideal of **S**. Let I_1 be any generalized interior Γ-hyperideal of **S** containing A. Then

$$\begin{aligned}\mathbf{S}\Gamma A\Gamma\mathbf{S} &\subseteq \mathbf{S}\Gamma I_1\Gamma\mathbf{S}\\ &\subseteq I_1.\end{aligned}$$

It implies $A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S} \subseteq I_1$ and hence I is the smallest generalized interior Γ-hyperideal of **S** containing A. Thus, $< A >_{i_{gen}} = I = A \cup \mathbf{S}\Gamma A\Gamma\mathbf{S}$. □

Theorem 3.4. *For a non-empty subset A of a Γ-semihypergroup* **S**,

$$< A >_{b_{gen}} = A \cup A\Gamma\mathbf{S}\Gamma A.$$

Moreover, if $A = \{a\}$, then $< \{a\} >_{i_{gen}} = a \cup \mathbf{S}\Gamma a\Gamma\mathbf{S}$ and $< \{a\} >_{b_{gen}} = a \cup a\Gamma\mathbf{S}\Gamma a$. We write $< \{a\} >_{i_{gen}}$ as $< a >_{i_{gen}}$ and $< \{a\} >_{b_{gen}}$ as $< a >_{b_{gen}}$.

Definition 3.5. A non-null soft set $\mathcal{F}_S$ is said to be an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup **S** over $\mathcal{V}$ if

$$\bigcap_{\vartheta \in x\Gamma y\Gamma z} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(y)\ \forall\ x,\ y,\ z \in \mathbf{S}.$$

Example 3.6. A grocery store is giving some offers for its products which can be defined in a set

$$\mathbf{S} = \{Soap,\ Toothpaste,\ 1\ Kg\ Sugar,\ 1\ Kg\ Tea\}.$$

The offer is defined by the following multiplication table:

∘	Soap	Toothpaste	1 Kg Sugar	1 Kg Tea
Soap	Soap	Soap	Soap	Soap
Toothpaste	Soap	Soap	Soap	Soap
1 Kg Sugar	Soap	Soap	Soap	{Soap, Toothpaste}
1 Kg Tea	Soap	Soap	{Soap, Toothpaste}	{Soap, Toothpaste, 1 Kg Sugar}

The hyperoperation defined in the above composition table as:
$x \circ y$ = On purchasing an $'x'$ item and an $'y'$ item from the grocery store, buyer can get $'X'$ items absolutely free, where $x,\ y \in \mathbf{S}$ and $X \subseteq \mathbf{S}$. Then $(\mathbf{S}, \Gamma)$ will be a Γ-semihypergroup, where $\Gamma = \{'\circ'\}$.
Let $\mathcal{V} = \{P_1,\ P_2,\ P_3\}$ be the set of peoples who have come to buy some products in grocery store. Define a soft set $\mathcal{F}_S : \mathbf{S} \to P(\mathcal{V})$ by

$\mathcal{F}_S(Soap) = \{P_1,\ P_2,\ P_3\}$, denotes the peoples who got one soap absolutely free

$\mathcal{F}_S(Toothpaste) = \{P_1,\ P_2\}$, denotes the peoples who got one toothpaste absolutely free

$\mathcal{F}_S(Sugar) = \{P_3\}$, denotes the peoples who got 1 Kg sugar absolutely free and

$\mathcal{F}_S(Tea) = \emptyset$, denotes the peoples who got 1 Kg tea absolutely free.

Thus, we can verify that $\bigcap_{\vartheta \in x\Gamma y\Gamma z} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(y)\ \forall\ x,\ y,\ z \in \mathbf{S}$. In particular, take $x = Sugar$, $y = Tea$ and $z = Soap$. Then, we have

$$\begin{aligned} x\Gamma y\Gamma z &= Sugar\Gamma Tea\Gamma Soap \\ &= Sugar \circ Tea \circ Soap \\ &= \{Soap, Toothpaste\} \circ Soap \\ &= (Soap \circ Soap) \cup (Toothpaste \circ Soap) \\ &= Soap \cup Soap \\ &= Soap. \end{aligned}$$

It implies $\bigcap_{\vartheta \in Sugar\Gamma Tea\Gamma Soap} \mathcal{F}_S(\vartheta) = \mathcal{F}_S(Soap) = \{P_1,\ P_2,\ P_3\} \supseteq \mathcal{F}_S(Tea) = \emptyset$. Therefore $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$ but $\mathcal{F}_S$ is not an S.I. Γ-hyperideal because $\bigcap_{\vartheta \in sugar\ o\ tea} \mathcal{F}_S(\vartheta) = \mathcal{F}_S(soap) \bigcap \mathcal{F}_S(toothpaste) = \{P_1,\ P_2\} \not\supseteq \mathcal{F}_S(sugar) = \{P_3\}$.

Definition 3.7. An S.I. generalized interior Γ-hyperideal $\mathcal{F}_S$ is said to be an S.I. interior Γ-hyperideal of a Γ-semihypergroup **S** over $\mathcal{V}$ if $\mathcal{F}_S$ is also an S.I. sub Γ-semihypergroup of **S** over $\mathcal{V}$.

Definition 3.8. A non-null soft set $\mathcal{F}_S$ is said to be an S.I. generalized bi-Γ-hyperideal of a Γ-semihypergroup **S** over $\mathcal{V}$ if

$$\bigcap_{\vartheta \in x\Gamma y\Gamma z} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(x) \cap \mathcal{F}_S(z)\ \forall\ x,\ y,\ z \in \mathbf{S}.$$

Example 3.9. A pathlab is giving two types of offers for couples(Husband and Wife) on some common tests, defined in a set
S = { LiverFunctionTest(KFT) KidneyFunctionTest(KFT), Cholesterol(Ch.), Glucose(Gl.), Hemoglobin(Hemo.)} with the following composition tables:

α	LFT	KFT	Ch.	Gl.	Hemo.
LFT	{KFT, Hemo.}	Hemo.	Ch.	{Ch., Gl.}	Hemo.
KFT	Hemo.	Hemo.	Ch.	{Ch., Gl.}	Hemo.
Ch.	Ch.	Hemo.	Ch.	Ch.	Hemo.
Gl.	{Ch., Gl.}	{Ch., Gl.}	Ch.	Gl.	{Ch., Gl.}
Hemo.	Hemo.	Hemo.	Ch.	{Ch., Gl.}	Hemo.

β	LFT	KFT	Ch.	Gl.	Hemo.
LFT	{LFT, KFT }	{KFT, Hemo.}	Ch.	{Ch., Gl.}	Hemo.
KFT	{ KFT, Hemo. }	Hemo.	Ch.	{Ch., Gl.}	Hemo.
Ch.	Ch.	Ch.	Ch.	Ch.	Ch.
Gl.	{Ch., Gl.}	{Ch., Gl.}	Ch.	Gl.	{Ch., Gl.}
Hemo.	Hemo.	Hemo.	Ch.	{Ch., Gl.}	Hemo.

Let H = 'Husband' and W = 'Wife'. Then the hyperoperations α and β defined in the above composition tables as:

(x α y) = If the pathlab does x test on H and y test on W, then the lab will give 50 percent discount on X tests to H, where $x, y \in \mathbf{S}$ and $X \subseteq \mathbf{S}$.
i.e. LFT α KFT = If the pathlab does LFT test on H and KFT test on W, then the lab will give 50 percent discount on $\{KFT, Hemo.\}$ tests to H.

(x β y) = If the pathlab does x test on H and y test on W, then the lab will give 50 percent discount on X tests to W, where $x, y \in \mathbf{S}$ and $X \subseteq \mathbf{S}$.
i.e. LFT β KFT = If the pathlab does LFT test on H and KFT test on W, then the lab will give 50 percent discount on $\{LFT, KFT\}$ tests to W.
Therefore, ($\mathbf{S}$, Γ) will be a Γ-semihypergroup, where $\Gamma = \{\alpha, \beta\}$. Now, let $\mathcal{U}$ = $\{C_1,\ C_2,\ C_3,\ C_4,\ C_5\}$ be the set of couples(Husband and Wife) who did this tests. Define a soft set $\mathcal{F}_{\mathbf{S}} : \mathbf{S} \to P(\mathcal{U})$ by

$\mathcal{F}_{\mathbf{S}}$(LFT) = $\{C_1,\ C_2\}$, denotes the couples who have done LFT test
$\mathcal{F}_{\mathbf{S}}$(KFT) = $\{C_1,\ C_2,\ C_3\}$, denotes the couples who have done KFT test
$\mathcal{F}_{\mathbf{S}}$(Ch.) = $\{C_1,\ C_2,\ C_3,\ C_4\}$, denotes the couples who have done Cholesterol test
$\mathcal{F}_{\mathbf{S}}$(Gl.) = $\{C_1,\ C_2,\ C_3,\ C_4,\ C_5\}$, denotes the couples who have done Glucose test(Sugar test) and
$\mathcal{F}_{\mathbf{S}}$(Hemo.) = $\{C_1,\ C_2\}$, denotes the couples who have done Hemoglobin test.

Then, we can verify that $\bigcap_{\vartheta \in x\Gamma y\Gamma z} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(x) \cap \mathcal{F}_S(z)\ \forall\ x,\ y,\ z \in \mathbf{S}$. Therefore, $\mathcal{F}_S$ is an S.I. generalized bi-Γ-hyperideal of $\mathbf{S}$ over $\mathcal{U}$.

Theorem 3.10. *A non-null soft set $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$ if and only if $\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{S}_S \sqsubseteq \mathcal{F}_S$.*

Proof. Suppose that $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$. Then, we have $\bigcap_{\vartheta \in x\Gamma y\Gamma z} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(y)\ \forall\ x,\ y,\ z \in \mathbf{S}$.
Now, if $\mathbb{S}_x = \emptyset$, then $(\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{S}_S)(x) = \emptyset$. It is clear that $(\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{S}_S)(x) \subseteq \mathcal{F}_S(x)$, therefore $\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{S}_S \sqsubseteq \mathcal{F}_S$.

If $\mathbb{S}_x \neq \emptyset$, then there exist $u,\ v \in \mathbf{S}$ such that $x \in u\ \Gamma\ v$ and $u \in p\ \Gamma\ q$. Hence, we have

$$\begin{aligned}
(\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{S}_S)(x) &= ((\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S) \mathbin{\hat{\diamond}} \mathcal{S}_S)(x) = \bigcup_{(u,v)\in \mathbb{S}_x} [(\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S)(u) \bigcap \mathcal{S}_S(v)] \\
&= \bigcup_{x\in u\Gamma v} [(\mathcal{S}_S \mathbin{\hat{\diamond}} \mathcal{F}_S)(u) \bigcap \mathcal{S}_S(v)] \\
&= \bigcup_{x\in u\Gamma v} [\bigcup_{(p,q)\in \mathbb{S}_u} (\mathcal{S}_S(p) \bigcap \mathcal{F}_S(q)) \bigcap \mathcal{S}_S(v)] \\
&= \bigcup_{x\in u\Gamma v} [\bigcup_{u\in p\Gamma q} (\mathcal{S}_S(p) \bigcap \mathcal{F}_S(q)) \bigcap \mathcal{S}_S(v)] \\
&= \bigcup_{x\in u\Gamma v} [\bigcup_{u\in p\Gamma q} (\mathcal{S}_S(p) \bigcap \mathcal{F}_S(q)) \bigcap \mathcal{V}] \\
&= \bigcup_{x\in u\Gamma v} [\bigcup_{u\in p\Gamma q} (\mathcal{S}_S(p) \bigcap \mathcal{F}_S(q))] = \bigcup_{x\in u\Gamma v} [\bigcup_{u\in p\Gamma q} (\mathcal{V} \bigcap \mathcal{F}_S(q))] \\
&= \bigcup_{x\in u\Gamma v} [\bigcup_{u\in p\Gamma q} (\mathcal{F}_S(q))] = \bigcup_{x\in (p\Gamma q)\Gamma v} (\mathcal{F}_S(q)) \\
&\subseteq \bigcup_{x\in (p\Gamma q)\Gamma v} \left\{ \bigcap_{\vartheta\in r\Gamma q\Gamma t} \mathcal{F}_S(\vartheta) \right\}, \subseteq \bigcup_{x\in (p\Gamma q)\Gamma v} \left\{ \bigcap_{x\in r\Gamma q\Gamma t} \mathcal{F}_S(x) \right\}
\end{aligned}$$

as $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S**

$$\begin{aligned} &\subseteq \bigcup_{x\in(p\Gamma q)\Gamma v}\left\{\bigcap_{x\in p\Gamma q\Gamma v}\mathcal{F}_S(x)\right\} \\ &= \mathcal{F}_S(x). \end{aligned}$$

Therefore, $\mathcal{S}_S \;\hat{\diamond}\; \mathcal{F}_S \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{F}_S$.

Conversely, suppose that $\mathcal{S}_S \;\hat{\diamond}\; \mathcal{F}_S \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{F}_S$. Then, we have to show that $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. Now, we have

$$\begin{aligned} \bigcap_{\vartheta\in x\Gamma y\Gamma z}\mathcal{F}_S(\vartheta) &\supseteq \bigcap_{\vartheta\in x\Gamma y\Gamma z}(\mathcal{S}_S \hat{\diamond} \mathcal{F}_S \hat{\diamond} \mathcal{S}_S)(\vartheta) \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}((\mathcal{S}_S \hat{\diamond} \mathcal{F}_S) \hat{\diamond} \mathcal{S}_S)(\vartheta) \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{(u,v)\in\$_\vartheta}\left[(\mathcal{S}_S \hat{\diamond} \mathcal{F}_S)(u)\bigcap\mathcal{S}_S(v)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[(\mathcal{S}_S \hat{\diamond} \mathcal{F}_S)(u)\bigcap\mathcal{S}_S(v)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{(p,q)\in\$_u}\left(\mathcal{S}_S(p)\bigcap\mathcal{F}_S(q)\right)\bigcap\mathcal{S}_S(v)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in p\Gamma q}\left(\mathcal{S}_S(p)\bigcap\mathcal{F}_S(q)\right)\bigcap\mathcal{S}_S(v)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in p\Gamma q}\left(\mathcal{S}_S(p)\bigcap\mathcal{F}_S(q)\right)\bigcap\mathcal{V}\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in p\Gamma q}\left(\mathcal{S}_S(p)\bigcap\mathcal{F}_S(q)\right)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in p\Gamma q}\left(\mathcal{V}\bigcap\mathcal{F}_S(q)\right)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in p\Gamma q}\left(\mathcal{F}_S(q)\right)\right]\right\} \\ &\supseteq \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma v}\left[\bigcup_{u\in x\Gamma y}\left(\mathcal{F}_S(y)\right)\right]\right\} \\ &\supseteq \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in u\Gamma z}\left[\bigcup_{u\in x\Gamma y}\left(\mathcal{F}_S(y)\right)\right]\right\} \\ &= \bigcap_{\vartheta\in x\Gamma y\Gamma z}\left\{\bigcup_{\vartheta\in(x\Gamma y)\Gamma z}\left(\mathcal{F}_S(y)\right)\right\} \\ &= \mathcal{F}_S(y). \end{aligned}$$

This implies $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. □

Theorem 3.11. *A non-null soft $\mathcal{F}_S$ is an S.I. sub Γ-semihypergroup of a Γ-semihypergroup* **S** *over $\mathcal{V}$ if and only if*

$$\mathcal{F}_S \;\hat{\diamond}\; \mathcal{F}_S \sqsubseteq \mathcal{F}_S.$$

Theorem 3.12. *A non-null soft $\mathcal{F}_S$ is an S.I. left Γ-hyperideal of a Γ-semihypergroup* **S** *over $\mathcal{V}$ if and only if*

$$\mathcal{S}_S \;\hat{\diamond}\; \mathcal{F}_S \sqsubseteq \mathcal{F}_S.$$

Theorem 3.13. *A non-null soft set $\mathcal{F}_S$ is an S.I. right Γ-hyperideal of a Γ-semihypergroup* **S** *over $\mathcal{V}$ if and only if*

$$\mathcal{F}_S \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{F}_S.$$

Theorem 3.14. *A non-null soft set $\mathcal{F}_S$ is an S.I. generalized bi Γ-hyperideal of a Γ-semihypergroup* **S** *over $\mathcal{V}$ if and only if*

$$\mathcal{F}_S \;\hat{\diamond}\; \mathcal{S}_S \;\hat{\diamond}\; \mathcal{F}_S \sqsubseteq \mathcal{F}_S.$$

Theorem 3.15. *If $\mathcal{F}_S$ and $\mathcal{G}_S$ are two S.I. generalized interior Γ-hyperideals of a Γ-semihypergroup* **S** *over $\mathcal{V}$. Then $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S$ is also an S.I. generalized interior Γ-hyperideal of* **S** *over $\mathcal{V}$.*

Proof. Suppose that $\mathcal{F}_S$ and $\mathcal{G}_S$ are two S.I. generalized interior Γ-hyperideals of a Γ-semihypergroup **S** over $\mathcal{V}$. Then, we have

$$\begin{aligned} \mathcal{S}_S \,\hat{\diamond}\, (\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{S}_S &\sqsubseteq \mathcal{S}_S \,\hat{\diamond}\, \mathcal{F}_S \,\hat{\diamond}\, \mathcal{S}_S \\ &\sqsubseteq \mathcal{F}_S \end{aligned}$$

and

$$\begin{aligned} \mathcal{S}_S \,\hat{\diamond}\, (\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{S}_S &\sqsubseteq \mathcal{S}_S \,\hat{\diamond}\, \mathcal{G}_S \,\hat{\diamond}\, \mathcal{S}_S \\ &\sqsubseteq \mathcal{G}_S \end{aligned}$$

It implies, $\mathcal{S}_S \;\hat{\diamond}\; (\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S) \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S$. Hence, $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. □

Theorem 3.16. *Every S.I. Γ-hyperideal of a Γ-semihypergroup* **S** *over $\mathcal{V}$ is an S.I. generalized interior Γ-hyperideal of a Γ-semhypergroup* **S** *over $\mathcal{V}$.*

Proof. Let $\mathcal{F}_S$ be an S.I. Γ-hyperideal of a Γ-semihypergroup **S** over $\mathcal{V}$. Then $\mathcal{S}_S \;\hat{\diamond}\; \mathcal{F}_S \sqsubseteq \mathcal{F}_S$ and $\mathcal{F}_S \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{F}_S$. Now

$$\begin{aligned} \mathcal{S}_S \,\hat{\diamond}\, \mathcal{F}_S \,\hat{\diamond}\, \mathcal{S}_S &= (\mathcal{S}_S \,\hat{\diamond}\, \mathcal{F}_S) \,\hat{\diamond}\, \mathcal{S}_S \\ &\sqsubseteq \mathcal{F}_S \,\hat{\diamond}\, \mathcal{S}_S \\ &\sqsubseteq \mathcal{F}_S. \end{aligned}$$

Hence, $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. □

Theorem 3.17. *If $\mathcal{F}_S$ and $\mathcal{G}_S$ are S.I. left and S.I. right Γ-hyperideals of a Γ-semihypergroup* **S** *over $\mathcal{V}$. Then, the S.I. product $\mathcal{F}_S \;\hat{\diamond}\; \mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of* **S** *over $\mathcal{V}$.*

Proof. Let $\mathcal{F}_S$ and $\mathcal{G}_S$ be the S.I. left and S.I. right Γ-hyperideals of a Γ-semihypergroup **S** over $\mathcal{V}$. Then $\mathcal{S}_S \,\hat{\diamond}\, \mathcal{F}_S \sqsubseteq \mathcal{F}_S$ and $\mathcal{G}_S \;\hat{\diamond}\; \mathcal{S}_S \sqsubseteq \mathcal{G}_S$. Now, we have

$$\begin{aligned} \mathcal{S}_S \,\hat{\diamond}\, (\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{S}_S &= (\mathcal{S}_S \,\hat{\diamond}\, (\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S)) \,\hat{\diamond}\, \mathcal{S}_S) \\ &= ((\mathcal{S}_S \,\hat{\diamond}\, \mathcal{F}_S) \,\hat{\diamond}\, \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{S}_S) \\ &\sqsubseteq (\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S) \,\hat{\diamond}\, \mathcal{S}_S, \text{ as } \mathcal{F}_S \text{ is an S.I. left } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V} \\ &= \mathcal{F}_S \,\hat{\diamond}\, (\mathcal{G}_S \,\hat{\diamond}\, \mathcal{S}_S) \\ &\sqsubseteq \mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S, \text{ as } \mathcal{G}_S \text{ is an S.I. right } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V}. \end{aligned}$$

It follows that $\mathcal{F}_S \;\hat{\diamond}\; \mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. □

Now, it is easy to prove the following proposition.

Proposition 3.18. *For any S.I. sub-Γ-semihypergroup $\mathcal{F}_S$, $\mathcal{S}_S \ \hat{\diamond}\ \mathcal{F}_S \ \hat{\diamond}\ \mathcal{S}_S$ is an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$.*

Proposition 3.19. *If $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$. Then, $\mathcal{S}_S \ \hat{\diamond}\ \mathcal{F}_S$ and $\mathcal{F}_S \ \hat{\diamond}\ \mathcal{S}_S$ are S.I. generalized interior Γ-hyperideals of $\mathbf{S}$ over $\mathcal{V}$.*

Theorem 3.20. *Let X be any non-empty subset of a Γ-semihypergroup $\mathbf{S}$. Then X is a generalized interior Γ-hyperideal of $\mathbf{S}$ if and only if its characteristic soft function $\mathcal{S}_X$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$.*

Proof. Let X be an interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$. Then, $\mathbf{S}\Gamma X\Gamma\mathbf{S} \subseteq X$. Now,

$$\begin{aligned} \mathcal{S}_S \ \hat{\diamond}\ \mathcal{S}_X \ \hat{\diamond}\ \mathcal{S}_S &= \mathcal{S}_{\mathbf{S}\Gamma X\Gamma\mathbf{S}} \\ &\sqsubseteq \mathcal{S}_X \end{aligned}$$

This shows that $\mathcal{S}_X$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. Conversely, suppose that $\mathcal{S}_X$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. Let $y \in \mathbf{S}\Gamma X\Gamma\mathbf{S}$, then

$$\begin{aligned} \mathcal{S}_X(y) &\supseteq \big(\mathcal{S}_S \ \hat{\diamond}\ \mathcal{S}_X \ \hat{\diamond}\ \mathcal{S}_S\big)(y) \\ &= \mathcal{S}_{\mathbf{S}\Gamma X\Gamma\mathbf{S}}(y) \\ &= \mathcal{V}. \end{aligned}$$

It implies $y \in X$. Hence, $\mathbf{S}\Gamma X\Gamma\mathbf{S} \subseteq X$. Therefore, X is a generalized interior Γ-hyperideal of $\mathbf{S}$. □

Theorem 3.21. *A non-null soft set $\mathcal{F}_S$ of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$ if and only if for each non-empty upper δ-inclusion of $\mathcal{F}_S$ is a generalized interior Γ-hyperideal of $\mathbf{S}$.*

Proof. Suppose that $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$ and upper δ-inclusion of $\mathcal{F}_S$, $\mathcal{V}(\mathcal{F}_S\ ;\ \delta) \neq \emptyset$, where $\delta \subseteq \mathcal{V}$. Let $a \in \mathbf{S}\ \Gamma\ \mathcal{V}(\mathcal{F}_S\ ;\ \delta)\ \Gamma\ \mathbf{S}$. Then $a \in u\ \Gamma\ b\ \Gamma\ v$ for some u, $v \in \mathbf{S}$ and $b \in \mathcal{V}(\mathcal{F}_S\ ;\ \delta)$. Now, $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. We have $\bigcap\limits_{\vartheta \in u\ \Gamma\ b\ \Gamma\ v} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(b)$. As, $a \in u\ \Gamma\ b\ \Gamma\ v$ it implies $\mathcal{F}_S(a) \supseteq \delta$. Thus $a \in \mathcal{V}(\mathcal{F}_S\ ;\ \delta)$ and hence, $\mathbf{S}\ \Gamma\ \mathcal{V}(\mathcal{F}_S\ ;\ \delta)\ \Gamma\ \mathbf{S} \subseteq \mathcal{V}(\mathcal{F}_S\ ;\ \delta)$. Therefore, $\mathcal{V}(\mathcal{F}_S\ ;\ \delta)$ is a generalized interior Γ-hyperideal of $\mathbf{S}$.
Conversely, suppose each non-empty upper δ-inclusion of $\mathcal{F}_S$ is a generalized interior Γ-hyperideal of $\mathbf{S}$, for every $\delta \subseteq \mathcal{V}$. Then we have to prove that $\bigcap\limits_{\vartheta \in u\ \Gamma\ a\ \Gamma\ v} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(a)$ for all a, u, $v \in \mathbf{S}$. Assume that $\mathcal{F}_S(a) = \delta$, then $a \in \mathcal{V}(\mathcal{F}_S\ ;\ \delta)$. Now, $u\ \Gamma\ a\ \Gamma\ v \subseteq \mathcal{V}(\mathcal{F}_S\ ;\ \delta)$, since $\mathcal{V}(\mathcal{F}_S\ ;\ \delta)$ is an interior Γ-hyperideal of $\mathbf{S}$. Then, for all $\vartheta \in u\ \Gamma\ a\ \Gamma\ v$, we have $\mathcal{F}_S(\vartheta) \supseteq \delta$ and

hence, $\mathcal{F}_S(a) = \delta \subseteq \bigcap_{\vartheta \in u\ \Gamma\ a\ \Gamma\ v} \mathcal{F}_S(\vartheta)$. Hence, $\mathcal{F}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. □

4. Characterizations of regular and intra-regular Γ-semihypergroups

In this section, we characterize regular and intra-regular Γ-Semihypergroups using S.I. generalized interior Γ-hyperideals and prove that for intra-regular Γ-semihypergroups, every S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$ is a constant function.

Theorem 4.1. *For a regular* Γ*-semihypergroup* **S***, every S.I. generalized interior* Γ*-hyperideal of* **S** *over* $\mathcal{V}$ *is an S.I. interior* Γ*-hyperideal of* **S** *over* $\mathcal{V}$*.*

Proof. Let **S** be a regular Γ-semihypergroup and $\mathcal{F}_S$ an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. As **S** is regular, it implies for any $a \in \mathbf{S}$ there exists $x \in \mathbf{S}$ such that $a \in a\ \Gamma\ x\ \Gamma\ a$. Thus for any $b \in \mathbf{S}$, we have $a\Gamma b \subseteq a\Gamma x\Gamma a\Gamma b$. Now,

$$\begin{aligned} \bigcap_{\vartheta \in a\Gamma b} \mathcal{F}_S(\vartheta) &\supseteq \bigcap_{\vartheta \in a\Gamma x\Gamma a\Gamma b} \mathcal{F}_S(\vartheta) \\ &= \bigcap_{\vartheta \in (a\Gamma x)\Gamma a\Gamma b} \mathcal{F}_S(\vartheta) \\ &\supseteq \mathcal{F}_S(a),\ \text{as } \mathcal{F}_{\mathrm{S}} \text{ is an S.I. generalized interior } \Gamma-\text{hyperideal} \\ &\supseteq \mathcal{F}_S(a)\ \cap\ \mathcal{F}_S(b). \end{aligned}$$

It implies, $\mathcal{F}_S$ is an S.I. interior Γ-hyperideal of **S** over $\mathcal{V}$.

□

Theorem 4.2. *For a regular* Γ*-semihypergroup* **S***, every S.I. generalized interior* Γ*-hyperideal of* **S** *over* $\mathcal{V}$ *is an S.I.* Γ*-hyperideal of* **S** *over* $\mathcal{V}$*.*

Proof. Proof is similar as given Theorem 4.1. □

Corollary 4.3. *For a regular* Γ*-semihypergroup* **S***, every S.I. interior* Γ*-hyperideal of* **S** *over* $\mathcal{V}$ *is an S.I.* Γ*-hyperideal of* **S** *over* $\mathcal{V}$*.*

Theorem 4.4. *For a* Γ*-semihypergroup* **S***, the following conditions are equivalent:*
(1) **S** *is regular;*
(2) $R \cap I \cap L \subseteq R\Gamma I\Gamma L$ *for every right* Γ*-hyperideal* R*, for every generalized interior* Γ*-hyperideal* I *and for every left* Γ*-hyperideal* L *of* **S***;*
(3) $< a >_r \cap < b >_{i_{gen}} \cap < c >_l \subseteq < a >_r \Gamma < b >_{i_{gen}} \Gamma < c >_l$ *for all* $a, b, c \in \mathbf{S}$*;*
(4) $< a >_r \cap < a >_{i_{gen}} \cap < a >_l \subseteq < a >_r \Gamma < a >_{i_{gen}} \Gamma < a >_l$ *for all* $a \in \mathbf{S}$.

Proof. Suppose that $\mathbf{S}$ is a regular Γ-semihypergroup and R, I, L are left, generalized interior and left Γ-hyperideal of $\mathbf{S}$, respectively. As $\mathbf{S}$ is regular, thus for any $a \in \mathbf{S}$, there exists $x \in \mathbf{S}$ such that $a \in a\Gamma x\Gamma a$. Now,

$$\begin{aligned} a &\in a\Gamma x\Gamma a \\ &\subseteq a\Gamma x\Gamma a\Gamma x\Gamma a \\ &\subseteq a\Gamma x\Gamma a\Gamma x\Gamma a\Gamma x\Gamma a \\ &= (a\Gamma x\Gamma a)\Gamma(x\Gamma a\Gamma x)\Gamma a. \end{aligned} \tag{4.1}$$

Let $a \in R \ \cap \ I \ \cap \ L$. Then, from (4.1)

$$\begin{aligned} a &\in (a\Gamma x\Gamma a)\Gamma(x\Gamma a\Gamma x)\Gamma a \\ &\subseteq (R\Gamma\mathbf{S}\Gamma\mathbf{S})\Gamma(\mathbf{S}\Gamma I\Gamma\mathbf{S})\Gamma L \\ &\subseteq (R\Gamma\mathbf{S})\Gamma(\mathbf{S}\Gamma I\Gamma\mathbf{S})\Gamma L \\ &\subseteq R\Gamma I\Gamma L. \end{aligned}$$

It implies $R \ \cap \ I \ \cap \ L \subseteq R\Gamma I\Gamma L$.
(2) $\Rightarrow$ (3) and (3) $\Rightarrow$ (4) is obvious.
(4) $\Rightarrow$ (1)
Suppose that (4) holds. To show $\mathbf{S}$ is regular, we have

$$\begin{aligned} a &\in \ < a >_r \ \cap \ < a >_{i_{gen}} \ \cap \ < a >_l \\ &\subseteq \ < a >_r \Gamma < a >_{i_{gen}} \Gamma < a >_l \\ &= \big(a \cup (a\Gamma\mathbf{S})\big)\Gamma\big(a \cup (\mathbf{S}\Gamma a\Gamma\mathbf{S})\big)\Gamma\big(a \cup (\mathbf{S}\Gamma a)\big) \\ &\subseteq (a\Gamma a\Gamma a) \cup (a\Gamma\mathbf{S}\Gamma a). \end{aligned}$$

It would imply that $a \in a\Gamma a\Gamma a$ or $a\Gamma\mathbf{S}\Gamma a$. Therefore, $\mathbf{S}$ is a regular Γ-semihypergroup. □

Theorem 4.5. *Let $\mathbf{S}$ be a Γ-semihypergroup. Then the following conditions are equivalent:*
(1) $\mathbf{S}$ is regular;
(2) $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \widetilde{\bigcap} \mathcal{H}_S \sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{H}_S$, for every S.I. generalized bi Γ-hyperideal $\mathcal{F}_S$, $\mathcal{H}_S$ of $\mathbf{S}$ over $\mathcal{V}$ and every S.I. generalized interior Γ-hyperideal $\mathcal{G}_S$ of $\mathbf{S}$ over $\mathcal{V}$;
(3) $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \widetilde{\bigcap} \mathcal{H}_S \sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{H}_S$, for every S.I. right Γ-hyperideal $\mathcal{F}_S$ of $\mathbf{S}$ over $\mathcal{V}$, every S.I. generalized interior Γ-hyperideal $\mathcal{G}_S$ of $\mathbf{S}$ over $\mathcal{V}$ and for every S.I. left Γ-hyperideal $\mathcal{H}_S$ of $\mathbf{S}$ over $\mathcal{V}$.

Proof. Let $\mathbf{S}$ be a regular Γ-semihypergroup. Then for any $a \in \mathbf{S}$, there exists $x \in \mathbf{S}$ such that $a \in a\Gamma x\Gamma a$. Now

$$\begin{aligned} a &\in a\Gamma x\Gamma a \\ &\subseteq a\Gamma x\Gamma a\Gamma x\Gamma a \\ &\subseteq a\Gamma x\Gamma a\Gamma x\Gamma a\Gamma x\Gamma a\Gamma x\Gamma a \\ &= (a\Gamma x\Gamma a)\Gamma(x\Gamma a\Gamma x)\Gamma(a\Gamma x\Gamma a). \end{aligned}$$

Then, there exists $u \in a\Gamma x\Gamma a$, $v \in x\Gamma a\Gamma x$ and $w \in a\Gamma x\Gamma a$ such that $a \in u\Gamma v\Gamma w$ and again there exists $m \in v\Gamma w$ such that $a \in u\Gamma m$. Hence, $\$_a$ and $\$_m$ are non-empty. Now, we have

$$\begin{aligned}\big(\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S\big)(a) &= \bigcup_{(y,z)\in \$_a} \big[\mathcal{F}_S(y) \cap \big(\mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S\big)(z)\big] \\ &\supseteq \mathcal{F}_S(u) \cap \big(\mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S\big)(m) \\ &= \mathcal{F}_S(u) \cap \Big\{ \bigcup_{(p,q)\in \$_m} \big[\mathcal{G}_S(p) \cap \mathcal{H}_S(q)\big]\Big\} \\ &\supseteq \mathcal{F}_S(u) \cap \mathcal{G}_S(v) \cap \mathcal{H}_S(w). \qquad (4.2)\end{aligned}$$

As $\mathcal{F}_S$ and $\mathcal{H}_S$ are S.I. generalized bi Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$, we have $\bigcap_{\vartheta\in a\Gamma x\Gamma a} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(a)$ and $\bigcap_{\theta\in a\Gamma x\Gamma a} \mathcal{H}_S(\theta) \supseteq \mathcal{H}_S(a)$. Since, $u \in a\Gamma x\Gamma a$ and $w \in a\Gamma x\Gamma a$, we have $\mathcal{F}_S(u) \supseteq \mathcal{F}_S(a)$ and $\mathcal{H}_S(w) \supseteq \mathcal{H}_S(a)$.
As $\mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$, we have $\bigcap_{\vartheta\in x\Gamma a\Gamma x} \mathcal{G}_S(\vartheta) \supseteq \mathcal{G}_S(a)$. Since, $v \in x\Gamma a\Gamma x$, we have $\mathcal{G}_S(v) \supseteq \mathcal{G}_S(a)$. Hence, from (4.2)

$$\begin{aligned}\big(\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S\big)(a) &\supseteq \mathcal{F}_S(u) \cap \mathcal{G}_S(v) \cap \mathcal{H}_S(w) \\ &\supseteq \mathcal{F}_S(a) \cap \mathcal{G}_S(a) \cap \mathcal{H}_S(a) \\ &= \big(\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \widetilde{\bigcap} \mathcal{H}_S\big)(a).\end{aligned}$$

Therefore, $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \widetilde{\bigcap} \mathcal{H}_S \sqsubseteq \mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S \,\hat{\diamond}\, \mathcal{H}_S$.
(2) ⇒ (3) is obvious.
(3) ⇒ (1)
Suppose that the condition (3) holds. To prove $\mathbf{S}$ is regular, we have to show that $R \cap I \cap L \subseteq R\Gamma I\Gamma L$ for every right Γ-hyperideal R, for every interior Γ-hyperideal I and for every left Γ-hyperideal L of $\mathbf{S}$. Let R, I and L be right, generalized interior and left Γ-hyperideal of $\mathbf{S}$, respectively. Then by Theorem 3.20, the soft characteristic functions $\mathcal{S}_R$, $\mathcal{S}_I$ and $\mathcal{S}_L$ will be S.I. right, S.I. generalized interior and S.I. left Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. By assumption, $\mathcal{S}_R \widetilde{\bigcap} \mathcal{S}_I \widetilde{\bigcap} \mathcal{S}_L \sqsubseteq \mathcal{S}_R \,\hat{\diamond}\, \mathcal{S}_I \,\hat{\diamond}\, \mathcal{S}_L$. Let $a \in R \cap I \cap L$. Then, $\mathcal{S}_{R\cap I\cap L} = \mathcal{V}$. Now, we have

$$\begin{aligned}\big(\mathcal{S}_{R\Gamma I\Gamma L}\big)(a) &= \big(\mathcal{S}_R \,\hat{\diamond}\, \mathcal{S}_I \,\hat{\diamond}\, \mathcal{S}_L\big)(a) \\ &\supseteq \big(\mathcal{S}_R \widetilde{\bigcap} \mathcal{S}_I \widetilde{\bigcap} \mathcal{S}_L\big)(a) \\ &= \mathcal{S}_{R\cap I\cap L}(a) \\ &= \mathcal{V}.\end{aligned}$$

It implies $a \in R\Gamma I\Gamma L$. Hence, $R \cap I \cap L \subseteq R\Gamma I\Gamma L$. Therefore by Theorem 4.4, $\mathbf{S}$ is regular. □

Theorem 4.6. *If* $\mathbf{S}$ *is a regular* Γ*-semihypergroup. Then for every S.I. generalized bi* Γ*-hyperideal* $\mathcal{F}_S$ *of* $\mathbf{S}$ *over* $\mathcal{V}$ *and every S.I. generalized interior* Γ*-hyperideal* $\mathcal{G}_S$ *of* $\mathbf{S}$ *over* $\mathcal{V}$*,*

$$\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S = \mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S \,\hat{\diamond}\, \mathcal{F}_S.$$

Proof. Let $\mathcal{F}_S$ be an S.I. generalized bi Γ-hyperideal of a Γ-semihypergroup **S** over $\mathcal{V}$ and $\mathcal{G}_S$ an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. Then

$$\begin{aligned}\mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S &\sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{S}_S \hat{\diamond} \mathcal{F}_S \\ &\sqsubseteq \mathcal{F}_S, \quad \text{as } \mathcal{F}_S \text{ is a S.I. generalized bi } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V},\end{aligned}$$

and

$$\begin{aligned}\mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S &\sqsubseteq \mathcal{S}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{S}_S \\ &\sqsubseteq \mathcal{G}_S, \quad \text{as } \mathcal{G}_S \text{ is an S.I. generalized interior } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V}.\end{aligned}$$

It follows that $\mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S \sqsubseteq \mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S$. Now, we will prove that $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S$. Let $s \in \mathbf{S}$. As **S** is regular, there exists $x \in \mathbf{S}$ such that $s \in s\Gamma x\Gamma s$, $s \in s\Gamma x\Gamma s\Gamma x\Gamma s$. Now, there exists $b \in x\Gamma s\Gamma x\Gamma s$ such that $s \in s\Gamma b$ and then there exists $m \in x\Gamma s\Gamma x$ such that $b \in m\Gamma s$. Hence, $\mathbb{S}_s$ and $\mathbb{S}_b$ are non-empty. Thus, we have

$$\begin{aligned}\big(\mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S\big)(s) &= \big[\mathcal{F}_S \hat{\diamond} \big(\mathcal{G}_S \hat{\diamond} \mathcal{F}_S\big)\big](s) \\ &= \bigcup_{(u,v)\in\mathbb{S}_s} \big[\mathcal{F}_S(u) \cap \big(\mathcal{G}_S \hat{\diamond} \mathcal{F}_S\big)(v)\big] \\ &\supseteq \mathcal{F}_S(s) \cap \big(\mathcal{G}_S \hat{\diamond} \mathcal{F}_S\big)(b) \\ &= \mathcal{F}_S(s) \cap \left\{ \bigcup_{(p,q)\in\mathbb{S}_b} \mathcal{G}_S(p) \cap \mathcal{F}_S(q) \right\} \\ &\supseteq \mathcal{F}_S(s) \cap \big(\mathcal{G}_S(m) \cap \mathcal{F}_S(s)\big) \\ &= \mathcal{F}_S(s) \cap \mathcal{G}_S(m). \end{aligned} \tag{4.3}$$

As $\mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$, we have $\bigcap_{\vartheta\in x\Gamma s\Gamma x} \mathcal{G}_S(\vartheta) \supseteq \mathcal{G}_S(s)$. Since, $m \in x\Gamma s\Gamma x$, we have $\mathcal{G}_S(m) \supseteq \mathcal{G}_S(s)$. Hence, from (4.3)

$$\begin{aligned}\big(\mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S\big)(s) &\supseteq \mathcal{F}_S(s) \cap \mathcal{G}_S(m) \\ &\supseteq \mathcal{F}_S(s) \cap \mathcal{G}_S(s) \\ &= \left(\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S\right)(s).\end{aligned}$$

Therefore, $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S = \mathcal{F}_S \hat{\diamond} \mathcal{G}_S \hat{\diamond} \mathcal{F}_S$. □

Lemma 4.7. *If a Γ-semihypergroup* **S** *is an intra-regular. Then,* **S** *is a generalized interior simple Γ-semihypergroup.*

Proof. Suppose that a Γ-semihypergroup **S** is intra-regular. Then, for each $a \in \mathbf{S}$, there exist $x, y \in \mathbf{S}$ such that $a \in x\ \Gamma\ a\ \Gamma\ a\ \Gamma\ y$. Now $a \in x\ \Gamma\ a\ \Gamma\ a\ \Gamma\ y = (x\ \Gamma\ a)\ \Gamma\ a\ \Gamma\ y \subseteq \mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S}$. It implies $a \in \mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S}$. Therefore, $\mathbf{S} \subseteq \mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S}$. i.e. $\mathbf{S}\ \Gamma\ a\ \Gamma\ \mathbf{S} = \mathbf{S}$ for all $a \in \mathbf{S}$. Hence by Lemma 2.12, **S** is a generalized interior simple. □

Theorem 4.8. *Let* **S** *be a Γ-semihypergroup. Then* **S** *is a generalized interior simple Γ-semihypergroup if and only if every S.I. generalized interior Γ-hyperideal of* **S** *over* $\mathcal{V}$ *is a constant function.*

Proof. Let $\mathbf{S}$ be a generalized interior simple Γ-semihypergroup and $\mathcal{F}_S$ be an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. Then, we have $\mathbf{S}\ \Gamma\ s\ \Gamma\ \mathbf{S} = \mathbf{S}$ for all $s \in \mathbf{S}$, i.e. for any $a, b \in \mathbf{S}$, there exist $u, v, w, p \in \mathbf{S}$ such that $a \in u\ \Gamma\ b\ \Gamma\ w$ and $b \in w\ \Gamma\ a\ \Gamma\ p$. As $b \in w\ \Gamma\ a\ \Gamma\ p$, it implies

$$\begin{aligned}\mathcal{F}_S(b) &\supseteq \bigcap_{\vartheta \in w\Gamma a\Gamma p} \mathcal{F}_A(\vartheta) \\ &\supseteq \mathcal{F}_S(a), \text{ as } \mathcal{F}_{\mathrm{S}} \text{ is a S.I. interior } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V}.\end{aligned}$$

Hence, we have $\mathcal{F}_S(b) \supseteq \mathcal{F}_S(a)$. Also $a \in u\ \Gamma\ b\ \Gamma\ w$, it implies

$$\begin{aligned}\mathcal{F}_S(a) &\supseteq \bigcap_{\vartheta \in u\Gamma b\Gamma w} \mathcal{F}_S(\vartheta) \\ &\supseteq \mathcal{F}_S(b), \text{ as } \mathcal{F}_{\mathrm{S}} \text{ is an S.I. generalized interior } \Gamma-\text{hyperideal of } \mathbf{S} \text{ over } \mathcal{V}.\end{aligned}$$

Hence, we have $\mathcal{F}_S(a) \supseteq \mathcal{F}_S(b)$. Here a and b are arbitrary. Thus, for all a and b, $\mathcal{F}_S(a) = \mathcal{F}_S(b)$. This shows that $\mathcal{F}_S$ is a constant function.
Conversely, suppose that every S.I. generalized interior Γ-hyperideal of a Γ-semihypergroup $\mathbf{S}$ over $\mathcal{V}$ is a constant function. Let M be a generalized interior Γ-hyperideal of $\mathbf{S}$ and $s \in \mathbf{S}$. By Theorem 3.20, its characteristic soft function $\mathcal{S}_M$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. By hypothesis, $\mathcal{S}_M$ is a constant function, that is, $\mathcal{S}_M(s) = \mathcal{S}_M(m)$ for every $m \in \mathbf{S}$. Let $m \in M$. Then, $\mathcal{S}_M(s) = \mathcal{S}_M(m) = \mathcal{V}$. Hence we have $s \in M$. Thus, we obtain $\mathbf{S} \subseteq M$. Therefore $\mathbf{S}$ is a generalized interior simple Γ-semihypergroup. □

Theorem 4.9. *If $\mathbf{S}$ is an intra-regular Γ-semihypergroup. Then every S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ is a constant function.*

Proof. Let $\mathbf{S}$ be an intra-regular Γ-semihypergroup. Then, by Lemma 4.7, $\mathbf{S}$ will be a generalized interior simple Γ-semihypergroup. Now, then by Theorem 4.8, every S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$ will be a constant function. □

5. Characterizations of semisimple and right weakly regular Γ-semihypergroups

In this section, we give some characterizations of semisimple and right weakly regular Γ-semihypergroups using S.I. generalized interior Γ-hyperideals and S.I. generalized bi-Γ-hyperideals.

Definition 5.1. A Γ-semihypergroup $\mathbf{S}$ is said to be a semisimple Γ-semihypergroup if for every $a \in \mathbf{S}$ there exist $x, y, z \in \mathbf{S}$ such that $a \in x\Gamma a\Gamma y\Gamma a\Gamma z$.

Theorem 5.2. *If $\mathbf{S}$ is a semisimple Γ-semihypergroup then every S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$ is a S.I. Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$.*

Proof. Let $\mathbf{S}$ be a semisimple Γ-semihypergroup and $\mathcal{F}_S$ an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. As $\mathbf{S}$ is a semisimple Γ-semihypergroup, then for every $a \in \mathbf{S}$ there exist $x, y, z \in \mathbf{S}$ such that $a \in x\Gamma a\Gamma y\Gamma a\Gamma z$. This implies that $a\Gamma b \subseteq x\Gamma a\Gamma y\Gamma a\Gamma z\Gamma b$. Thus we have

$$\begin{aligned}\bigcap_{\vartheta\in a\Gamma b}\mathcal{F}_S(\vartheta) &\supseteq \bigcap_{\vartheta\in x\Gamma a\Gamma y\Gamma a\Gamma z\Gamma b}\mathcal{F}_S(\vartheta)\\ &= \bigcap_{\vartheta\in(x\Gamma a\Gamma y)\Gamma a\Gamma(z\Gamma b)}\mathcal{F}_S(\vartheta)\\ &\supseteq \mathcal{F}_S(a), \text{ as } \mathcal{F}_{\mathrm{S}} \text{ is an S.I. generalized interior } \Gamma-\text{hyperideal.}\end{aligned}$$

Analogously, we can prove that $\bigcap_{\vartheta\in a\Gamma b}\mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(b)$. Hence, every S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$ is a S.I. Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$. □

Theorem 5.3. *Let* $\mathbf{S}$ *be a* Γ*-semihypergroup. Then the following conditions are equivalent:*
(1) $\mathbf{S}$ *is semisimple;*
(2) $I_1 \bigcap I_2 \subseteq I_1\ \Gamma\ I_2$ *for every generalized interior* Γ*-hyperideals* I_1 *and* I_2 *of* $\mathbf{S}$*;*
(3) $I \subseteq I\ \Gamma\ I$ *for every generalized interior* Γ*-hyperideal* I *of* $\mathbf{S}$*;*
(4) $< a >_{i_{gen}} \subseteq < a >_{i_{gen}}\ \Gamma\ < a >_{i_{gen}}$ *for every* $a \in \mathbf{S}$.

Proof. Suppose that $\mathbf{S}$ is a semisimple Γ-semihypergroup and I_1, I_2 are the generalized interior Γ-hyperideal of $\mathbf{S}$. Let $a \in I_1 \bigcap I_2$. As $\mathbf{S}$ is a semisimple Γ-semihypergroup, then for any $a \in \mathbf{S}$ there exist $x, y, z \in \mathbf{S}$ such that $a \in x\Gamma a\Gamma y\Gamma a\Gamma z$. Now,

$$\begin{aligned} a &\in x\Gamma a\Gamma y\Gamma a\Gamma z\\ &\subseteq x\Gamma a\Gamma y\Gamma((x\Gamma a\Gamma y)\Gamma a\Gamma z\Gamma z)\\ &\subseteq (\mathbf{S}\Gamma I_1\Gamma\mathbf{S})\Gamma((\mathbf{S}\Gamma I_1\Gamma\mathbf{S})\Gamma I_2\Gamma(\mathbf{S}\Gamma\mathbf{S}))\\ &\subseteq I_1\Gamma(I_1\Gamma I_2\Gamma\mathbf{S})\\ &\subseteq I_1\Gamma(\mathbf{S}\Gamma I_2\Gamma\mathbf{S})\\ &\subseteq I_1\Gamma I_2.\end{aligned}$$

Therefore, $I_1 \bigcap I_2 \subseteq I_1\ \Gamma\ I_2$.
Proofs of (2) ⇒ (3) and (3) ⇒ (4) are easy. We prove (4) ⇒ (1)
Suppose that (4) holds. To prove $\mathbf{S}$ is semisimple, let $a \in \mathbf{S}$. Then $a \in < a >_{i_{gen}}$, where $< a >_{i_{gen}}$ is a generalized interior Γ-hyperideal of $\mathbf{S}$ containing a. Now, we have

$$\begin{aligned} a &\in < a >_{i_{gen}}\\ &\subseteq < a >_{i_{gen}}\ \Gamma\ < a >_{i_{gen}}\\ &= \big(a \cup (\mathbf{S}\Gamma a\Gamma\mathbf{S})\big)\ \Gamma\ \big(a \cup (\mathbf{S}\Gamma a\Gamma\mathbf{S})\big).\end{aligned}$$

Thus, $a \in a\Gamma a$ or $a \in (\mathbf{S}\Gamma a\Gamma\mathbf{S}\Gamma a)$ or $a \in (a\Gamma\mathbf{S}\Gamma a\Gamma\mathbf{S})$ or $a \in (\mathbf{S}\Gamma a\Gamma\mathbf{S}\Gamma a\Gamma\mathbf{S})$.
Hence $a \in a\Gamma a \subseteq a\Gamma a\Gamma a \subseteq a\Gamma a\Gamma a\Gamma a\Gamma a$ or
$a \in (x\Gamma a\Gamma y\Gamma a) \subseteq (x\Gamma a\Gamma y\Gamma x_1\Gamma a\Gamma y_1\Gamma a) = (x\Gamma a\Gamma(y\Gamma x_1)\Gamma a\Gamma(y_1\Gamma a))$. Therefore, there exists an element $z_1 \in y\Gamma x_1$ and $z_2 \in (y_1\Gamma a)$ such that $a \in (x\Gamma a\Gamma z_1\Gamma a\Gamma z_2)$ or
$a \in (a\Gamma x\Gamma a\Gamma y) \subseteq (a\Gamma x\Gamma a\Gamma x_1\Gamma a\Gamma y_1\Gamma y) = ((a\Gamma x)\Gamma a\Gamma x_1\Gamma a\Gamma(y_1\Gamma y))$. Therefore,

there exists an element $z_3 \in a\Gamma x$ and $z_4 \in (y_1\Gamma y)$ such that $a \in (z_3\Gamma a\Gamma x_1\Gamma a\Gamma z_4)$ or
$a \in (x\Gamma a\Gamma y\Gamma a\Gamma z)$. Hence it implies **S** is semisimple. □

Theorem 5.4. *Let* **S** *be a* Γ*-semihypergroup. Then the following conditions are equivalent:*
(1) **S** *is semisimple;*
(2) $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{G}_S$ *, for every S.I. generalized interior* Γ*-hyperideals* $\mathcal{F}_S$ *and* $\mathcal{G}_S$ *of* **S** *over* $\mathcal{V}$*.*

Proof. Let **S** be a semisimple Γ-semihypergroup and $\mathcal{F}_S$, $\mathcal{G}_S$ the S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$. As **S** is a semisimple Γ-semihypergroup, thus for any $a \in \mathbf{S}$, there exist $x, y, z \in \mathbf{S}$ such that $a \in x\Gamma a\Gamma y\Gamma a\Gamma z$ and $a \in x\Gamma a\Gamma y\Gamma a\Gamma z \subseteq x\Gamma a\Gamma y\Gamma(x\Gamma a\Gamma y\Gamma a\Gamma z)\Gamma z = (x\Gamma a\Gamma y)\Gamma x\Gamma a\Gamma(y\Gamma a\Gamma z\Gamma z)$. Thus, there exists $u \in x\Gamma a\Gamma y$, $v \in x\Gamma a\Gamma(y\Gamma a\Gamma z\Gamma z)$ such that $a \in u\Gamma v$. Hence, $\$_a$ is non-empty. Now, we have

$$\begin{aligned}\big(\mathcal{F}_S \hat{\diamond} \mathcal{G}_S\big)(a) &= \bigcup_{(p,q)\in\$_a} \big[\mathcal{F}_S(p) \cap \mathcal{G}_S(q)\big] \\ &\supseteq \mathcal{F}_S(u) \cap \mathcal{G}_S(v). \qquad (5.1)\end{aligned}$$

As $\mathcal{F}_S$, $\mathcal{G}_S$ are S.I. generalized interior Γ-hyperideal of **S** over $\mathcal{V}$, we have $\bigcap_{\vartheta\in x\Gamma a\Gamma y} \mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(a)$ and $\bigcap_{\theta\in x\Gamma a\Gamma(y\Gamma a\Gamma z\Gamma z)} \mathcal{G}_S(\theta) \supseteq \mathcal{G}_S(a)$. Since $u \in x\Gamma a\Gamma y$, we have $\mathcal{F}_S(u) \supseteq \mathcal{F}_S(a)$ and $v \in x\Gamma a\Gamma(y\Gamma a\Gamma z\Gamma z)$, we have $\mathcal{G}_S(v) \supseteq \mathcal{G}_S(a)$. Hence, from (5.1)

$$\begin{aligned}\big(\mathcal{F}_S \hat{\diamond} \mathcal{G}_S\big)(a) &\supseteq \mathcal{F}_S(u) \cap \mathcal{G}_S(v) \\ &\supseteq \mathcal{F}_S(a) \cap \mathcal{G}_S(a) \\ &= \big(\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S\big)(a).\end{aligned}$$

Therefore, $\mathcal{F}_S \widetilde{\bigcap} \mathcal{G}_S \sqsubseteq \mathcal{F}_S \hat{\diamond} \mathcal{G}_S$.
(2) ⇒ (1)
To show **S** is semisimple, we have to prove that $I_1 \bigcap I_2 \subseteq I_1 \,\Gamma\, I_2$ for every generalized interior Γ-hyperideals I_1 and I_2 of **S**. Suppose that the condition (2) holds and I_1, I_2 are the generalized interior Γ-hyperideals of **S**. Then by Theorem 3.20, the soft characteristic functions $\mathcal{S}_{I_1}$, $\mathcal{S}_{I_2}$ are S.I. generalized interior Γ-hyperideals of **S** over $\mathcal{V}$. By assumption, $\mathcal{S}_{I_1} \widetilde{\bigcap} \mathcal{S}_{I_2} \sqsubseteq \mathcal{S}_{I_1} \hat{\diamond} \mathcal{S}_{I_2}$. Let $a \in I_1 \cap I_2$. Then $\mathcal{S}_{I_1\cap I_2} = \mathcal{V}$. Now, we have

$$\begin{aligned}\big(\mathcal{S}_{I_1\Gamma I_2}\big)(a) &= \big(\mathcal{S}_{I_1} \hat{\diamond} \mathcal{S}_{I_2}\big)(a) \\ &\supseteq \big(\mathcal{S}_{I_1} \widetilde{\bigcap} \mathcal{S}_{I_2}\big)(a) \\ &= \mathcal{S}_{(I_1 \,\cap\, I_2)}(a) \\ &= \mathcal{V}\end{aligned}$$

It implies $a \in I_1\Gamma I_2$. Hence, $I_1 \bigcap I_2 \subseteq I_1 \,\Gamma\, I_2$. Therefore, **S** is semisimple. □

Definition 5.5. A Γ-semihypergroup $\mathbf{S}$ is said to be a right weakly regular Γ-semihypergroup if for every $a \in \mathbf{S}$ there exist $x, y \in \mathbf{S}$ such that $a \in a\Gamma x\Gamma a\Gamma y$.

Theorem 5.6. *Let $\mathbf{S}$ be a Γ-semihypergroup. Then the following conditions are equivalent:*
(1) $\mathbf{S}$ is right weakly regular;
(2) $B \bigcap I \subseteq B\ \Gamma\ I$ for every generalized bi Γ-hyperideal B and for every generalized interior Γ-hyperideal I of $\mathbf{S}$;
(3) $< a >_{b_{gen}} \cap < b >_{I_{gen}} \subseteq < a >_{b_{gen}}\ \Gamma\ < b >_{i_{gen}}$ for every $a, b \in \mathbf{S}$;
(4) $< a >_{b_{gen}} \cap < a >_{i_{gen}} \subseteq < a >_{b_{gen}}\ \Gamma\ < a >_{i_{gen}}$ for every $a \in \mathbf{S}$.

Proof. Suppose that $\mathbf{S}$ is a right weakly regular Γ-semihypergroup and B, I are the bi-Γ-hyperideal and interior Γ-hyperideal of $\mathbf{S}$ respectively. As $\mathbf{S}$ is a right weakly regular Γ-semihypergroup, thus for any $a \in \mathbf{S}$, there exist $x, y \in \mathbf{S}$ such that $a \in a\Gamma x\Gamma a\Gamma y$. Now,

$$\begin{aligned} a &\in a\Gamma x\Gamma a\Gamma y \\ &\subseteq a\Gamma x\Gamma(a\Gamma x\Gamma a\Gamma y)\Gamma y \\ &\subseteq (a\Gamma x\Gamma a)\Gamma x\Gamma a\Gamma(y\Gamma y). \end{aligned} \tag{5.2}$$

Let $a \in B\ \cap\ I$. Then from (5.2)

$$\begin{aligned} a &\in (a\Gamma x\Gamma a)\Gamma x\Gamma a\Gamma(y\Gamma y) \\ &\subseteq (B\Gamma\mathbf{S}\Gamma B)\Gamma(\mathbf{S}\Gamma I\Gamma(\mathbf{S}\Gamma\mathbf{S})) \\ &\subseteq B\Gamma(\mathbf{S}\Gamma I\Gamma\mathbf{S}) \\ &\subseteq B\Gamma I. \end{aligned}$$

It implies $B\ \cap\ I \subseteq B\Gamma I$.
(2) $\Rightarrow$ (3) and (3) $\Rightarrow$ (4) is obvious.
(4) $\Rightarrow$ (1)
Suppose that (4) holds. To show $\mathbf{S}$ is right weakly regular, we have

$$\begin{aligned} a &\in\ < a >_{b_{gen}}\ \cap\ < a >_{i_{gen}} \\ &\subseteq\ < a >_{b_{gen}} \Gamma < a >_{i_{gen}} \\ &= \big(a \cup (a\Gamma\mathbf{S}\Gamma a)\big)\Gamma\big(a \cup (\mathbf{S}\Gamma a\Gamma\mathbf{S})\big) \\ &\subseteq (a\Gamma a) \cup (a\Gamma\mathbf{S}\Gamma a\Gamma\mathbf{S}). \end{aligned}$$

It would imply that $a \in a\Gamma a$ or $a \in a\Gamma\mathbf{S}\Gamma a\Gamma\mathbf{S}$. Therefore, $\mathbf{S}$ is a right weakly regular Γ-semihypergroup. □

Theorem 5.7. *Let $\mathbf{S}$ be a Γ-semihypergroup. Then the following conditions are equivalent:*
(1) $\mathbf{S}$ is right weakly regular;
(2) $\mathcal{F}_S\ \widetilde{\bigcap}\ \mathcal{G}_S \sqsubseteq \mathcal{F}_S \mathbin{\hat{\diamond}} \mathcal{G}_S$, for every S.I. generalized bi Γ-hyperideal $\mathcal{F}_S$ of $\mathbf{S}$ over $\mathcal{V}$ and for every S.I. generalized interior Γ-hyperideal $\mathcal{G}_S$ of $\mathbf{S}$ over $\mathcal{V}$.

Proof. Let $\mathbf{S}$ be a right weakly regular Γ-semihypergroup and $\mathcal{F}_S$, $\mathcal{G}_S$ the S.I. generalized bi Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$, S.I. generalized interior Γ-hyperideal

$\mathcal{F}_S$ of $\mathbf{S}$ over $\mathcal{V}$ respectively. As $\mathbf{S}$ is a right weakly regular Γ-semihypergroup, then for any $a \in \mathbf{S}$, there exist x, $y \in \mathbf{S}$ such that $a \in a\Gamma x\Gamma a\Gamma y$. Thus $a \in a\Gamma x\Gamma a\Gamma y \subseteq a\Gamma x\Gamma(a\Gamma x\Gamma a\Gamma y)\Gamma y = (a\Gamma x\Gamma a)\Gamma(x\Gamma a\Gamma y\Gamma y)$. Then there exists $u \in a\Gamma x\Gamma a$ and $v \in x\Gamma a\Gamma(y\Gamma y)$ such that $a \in u\Gamma v$. Hence $\mathbb{S}_a$ is non-empty. Now we have

$$\begin{aligned}\big(\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S\big)(a) &= \bigcup_{(p,q)\in\mathbb{S}_a}\big[\mathcal{F}_S(p) \,\cap\, \mathcal{G}_S(q)\big]\\ &\supseteq \mathcal{F}_S(u) \,\cap\, \mathcal{G}_S(v). \qquad (5.3)\end{aligned}$$

As $\mathcal{F}_S$ is an S.I. generalized bi Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$, we have $\bigcap_{\vartheta\in a\Gamma x\Gamma a}\mathcal{F}_S(\vartheta) \supseteq \mathcal{F}_S(a)$ and $\mathcal{G}_S$ is an S.I. generalized interior Γ-hyperideal of $\mathbf{S}$ over $\mathcal{V}$, we have $\bigcap_{\theta\in x\Gamma a\Gamma(y\Gamma y)}\mathcal{G}_S(\theta) \supseteq \mathcal{G}_S(a)$. Since $u \in a\Gamma x\Gamma a$, we have $\mathcal{F}_S(u) \supseteq \mathcal{F}_S(a)$ and $v \in x\Gamma a\Gamma(y\Gamma y))$, we have $\mathcal{G}_S(v) \supseteq \mathcal{G}_S(a)$. Hence, from (5.3)

$$\begin{aligned}\big(\mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S\big)(a) &\supseteq \mathcal{F}_S(u) \,\cap\, \mathcal{G}_S(v)\\ &\supseteq \mathcal{F}_S(a) \,\cap\, \mathcal{G}_S(a)\\ &= \big(\mathcal{F}_S \,\widetilde{\bigcap}\, \mathcal{G}_S\big)(a).\end{aligned}$$

Therefore, $\mathcal{F}_S \,\widetilde{\bigcap}\, \mathcal{G}_S \sqsubseteq \mathcal{F}_S \,\hat{\diamond}\, \mathcal{G}_S$.

(2) ⇒ (1)

To show $\mathbf{S}$ is right weakly regular, we have to prove that $B \bigcap I \subseteq B\ \Gamma\ I$ for every generalized bi Γ-hyperideal B and generalized interior Γ-hyperideal I of $\mathbf{S}$. Suppose that the condition (2) holds and B, I are the generalized bi and generalized interior Γ-hyperideals of $\mathbf{S}$. Then by Theorem 3.20, the soft characteristic functions $\mathcal{S}_B$, $\mathcal{S}_I$ are S.I. generalized bi and S.I. generalized interior Γ-hyperideals of $\mathbf{S}$ over $\mathcal{V}$. By assumption, $\mathcal{S}_B \,\widetilde{\bigcap}\, \mathcal{S}_I \sqsubseteq \mathcal{S}_B \,\hat{\diamond}\, \mathcal{S}_I$. Let $a \in B \cap I$. Then $\mathcal{S}_{B\,\cap\,I} = \mathcal{V}$. Now, we have

$$\begin{aligned}\big(\mathcal{S}_{B\Gamma I}\big)(a) &= \big(\mathcal{S}_B \,\hat{\diamond}\, \mathcal{S}_I\big)(a)\\ &\supseteq \big(\mathcal{S}_B \,\widetilde{\bigcap}\, \mathcal{S}_I\big)(a)\\ &= \mathcal{S}_{(B\,\cap\,I)}(a)\\ &= \mathcal{V}\end{aligned}$$

It implies $a \in B\Gamma I$. Hence, $B \bigcap I \subseteq B\ \Gamma\ I$. Therefore, $\mathbf{S}$ is right weakly regular. □

6. Conclusion

In this paper, we have introduced soft intersection generalized interior-Γ-hyperideals and soft intersection generalized bi-Γ-hyperideals in Γ-semihypergroups. Moreover, some important characterizations of some classes of Γ-semihypergroups have been discussed in terms of soft intersection generalized interior-Γ-hyperideals and soft intersection generalized bi-Γ-hyperideals. Based on the results of this paper, some further work can be done on the properties of soft intersection generalized interior-Γ-hyperideals and soft intersection generalized bi-Γ-hyperideals in other hyperstructures.

Acknowledgments

We would like to express our warmest thanks to the referee for his time to read the manuscript very carefully and his useful sugestions.

References

1. S. Abdullah, K. Hila, M. Aslam, *On bi-Γ-hyperideals of Γ-semihypergroups*, *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.*, **74**(4), (2012), 79-90.
2. M. Y. Abbasi, K. Hila, S. A. Khan, A. F. Talee, Characterizations of Γ-hyperideals in Ordered Γ-semihypergroups by Soft Sets, *Afrika Matematika*, **31**(5-6), (2020), 847-867.
3. H. Aktas, N. Cagman, Soft Sets and Soft Groups, *Information Sci.*, **177**, (2007), 2726-2735.
4. S. M. Anvariyeh, S. Mirvakili, B. Davvaz, On Γ-hyperideals in Γ-semihypergroups, *Carpathian J. Math.*, **26**(1), (2010), 11–23.
5. S. M. Anvariyeh, S. Mirvakili, B. Davvaz, Pawlak's Approximations in Γ-semihypergroups, *Comput. Math. Appl.*, **60**, (2010), 45-53.
6. S. M. Anvariyeh, S. Mirvakili, O. Kazanci, B. Davvaz, Algebraic Hyperstructures of Soft Sets Associated to Semihypergroups, *Southeast Asian Bull. Math.*, **35**, (2011), 911-925.
7. M. Aslam, S. Abdullah, B. Davvaz, N. Yaqoob, Rough M-hypersystems and Fuzzy M-hypersystems in Γ-semihypergroups, *Neural Comput. Appl.*, **21**, (2012), 281-287.
8. N. Cagman, S. Enginoglu, Soft Set Theory and Uni-int Decision Making, *Eur. J. Op. Res.*, **207**, (2010), 848–-855.
9. N. Cagman, F. Citak, H. Aktas, Soft Int-group and Its Applications to Group Theory, *Neural Comput. Appl.*, **21**, (2012), 151-158.
10. J. Chvalina, *Commutative Hypergroups in the Sense of Marty and Ordered Sets*, General algebra and ordered sets (Horní Lipová, 1994), 19–30, Palacký Univ. Olomouc Fac . Sci., Olomouc, [1994].
11. P. Corsini, V. Leoreanu, *Applications of Hyperstructure Theory*, Advances in Mathematics (Dordrecht), 5. Kluwer Academic Publishers, Dordrecht, 2003. xii+322 pp. ISBN: 1-4020-1222-5.
12. P. Corsini, *Prolegomena of Hypergroup Theory*, Supplement to Riv. Mat. Pura Appl. Aviani Editore, Tricesimo, 1993. 215 pp. ISBN: 88-7772-025-5.
13. B. Davvaz, T. Vougiouklis, n-ary hypergroups, *Iran. J. Sci. Technol. Trans. A Sci.*, **30**, (2006), 165–174.
14. B. Davvaz, V. L. Fotea, *Hyperring Theory and Applications*, International Academic Press, Palm Harber, Fla, USA, (2007), 115.
15. B. Davvaz, V. Leoreanu-Fotea, Structures of Fuzzy Γ-hyperideals in Γ-semihypergroups, *J. Mult.-Valued Logic Soft Comput.*, **19**, (2012), 519–535.
16. B. Davvaz, V. Leoreanu-Fotea, Triangular Fuzzy Sub Γ-semihypergroups in Γ-semihypergroups, *Kuwait J. Sci.*, **40**(1), (2014), 93-106.
17. B. A. Ersoy, Y. Saricaoglu, M. Yenigun, B. Davvaz, On Fuzzy Interior Γ-hyperideals of Γ-semihypergroups, *Util. Math.*, **88** (2012), 157–170.
18. M. Farooq, A. Khan, B. Davvaz, Characterizations of Ordered Semihypergroups by the Properties of Their Intersectional-soft Generalized Bi-hyperideals, *Soft Comput.*, **22**, (2018), 3001-3010.

19. F. Feng, Y. B. Jun, X. Zhao, Soft Semirings, *Comput. Math. Appl.*, **56**, (2008), 2621–2628.
20. Y. Feng, P. Corsini, (λ, μ)-fuzzy Interior Ideals of Ordered Γ-semigroups, *Algebra Discrete Math.*, **16**(1), (2013), 61-70.
21. F. Feng, M. I. Ali, M. Shabir, Soft Relations Applied to Semigroups, *Filomat*, **27**(7), (2013), 1183-1196.
22. F. Feng, Y. Li, Soft Subsets and Soft Product Operations, *Information Sci.*, **232**, (2013), 44-57.
23. A. Hasankhani, Ideals in a Semihypergroup and Green's Relations, *Ratio Math.*, **13**, (1999), 29–36.
24. D. Heidari, S. O. Dehkordi, B. Davvaz, Γ-Semihypergroups and Their Properties, *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.*, **72**(1), (2010), 195-208.
25. D. Heidari, B. Davvaz, Γ-hypergroups and Γ-Semihypergroups Associated to Binary Relations, *Iran. J. Sci. Technol. Trans. A Sci.*, **35**(2), (2011), 69–80.
26. K. Hila, B. Davvaz, J. Dine, Study on the Structure of Γ-Semihypergroups, *Comm. Algebra*, **40**, (2012), 2932-2948.
27. S. Khademan, M. Z. Yazdi, Y. B. Jun, R. A. Borzooei, Fuzzy Soft Positive Implicative Hyper BCK-ideals in Hyper BCK-algebras, *J. Intell. Fuzzy Systems*, **36**(3), (2019), 2605-2613.
28. X. Liu, F. Feng, Y. B. Jun, A Note on Generalized Soft Equal Relations, *Comput. Math. Appl.*, **64**(4), (2012), 572-578.
29. X. Ma, J. Zhan, Characterizations of Three Kinds of Hemirings by Fuzzy Soft h-ideals, *J. Intell. Fuzzy Systems*, **24**, (2013), 535–548.
30. P. K. Maji, R. Biswas, A. R. Roy, Soft Set Theory, *Comput. Math. Appl.*, **45**, (2003), 555–562.
31. F. Marty, *Sur Une Generalization de la Notion de Group*, 8th Congres Math. Scandinaves Stockholm, 1934, 45–49.
32. G. Mohanraj , D. Krishnaswamy, R. Hema, On Generalized Fuzzy Weakly Interior Ideals of Ordered Semigroups, *Ann. Fuzzy Math. Inform.*, **8**(5), (2014), 803-814.
33. D. Molodtsov, Soft Set Theory-first Results, *Comput. Math. Appl.*, **37**, (1999), 19–31.
34. S. Naz, M. Shabir, On Soft Semihypergroups, *J. Intell. Fuzzy Systems*, **26**(5), (2014), 2203-2213.
35. S. K. Sardar, B. Davvaz, S. K. Majumdera, A Study on Fuzzy Interior Ideals of Γ-semigroups, *Comput. Math. Appl.* **60**(1), (2010), 90–94.
36. M. K. Sen, *On* Γ*-semigroups*, Algebra and its applications, Int. Symp., New Delhi, 1981, Lecture Notes in Pure and Applied Mathematics 91, Decker, New York, 1984, 301-308.
37. A. Sezgin, A. O. Atagun, On Operations of Soft Sets, *Comput. Math. Appl.*, **61**(5), (2011), 1457–1467.
38. A. Sezgin, N. Cagman, A. O. Atagun, Soft Intersection Interior Ideals, Quasi-ideals and Generalized bi-ideals; A New Approach to Semigroup Theory II, *J. Mult.-Valued Logic Soft Comput.*, **23**(1-2), (2014), 161–207.
39. A. Sezgin, N. Cagman, A. O. Atagun, M. Ali, Soft Intersection Semigroups, Ideals and Bi-ideals; a New Approach to Semigroup Theory, *Filomat*, **29**(5), (2015), 917–946
40. J. Tang , B. Davvaz, Y. Luo, A Study on Fuzzy Interior Hyperideals in Ordered Semihypergroups, *Ital. J. Pure Appl. Math.*, **36**, (2016), 125–146.
41. J. Tang, B. Davvaz, X.Y. Xie, N. Yaqoob, On Fuzzy Interior Γ-hyperideals in Ordered Γ-semihypergroups, *J. Intell. Fuzzy Systems*, **32**(3), (2017), 2447–2460.
42. N. Tipachot, B. Pibaljommee, Fuzzy Interior Hyperideals in Ordered Semihypergroups, *Ital. J. Pure Appl. Math.*, **36**, (2016), 859–870.

43. N. Yaqoob, M. Aslam, B. Davvaz, A. Ghareeb, Structures of Bipolar Fuzzy Γ-hyperideals in Γ-semihypergroups, *J. Intell. Fuzzy Systems*, **27**(6), (2014), 3015–3032.
44. J. Zhan, N. Cagman, A. S. Sezer, Applications of Soft Union Sets to Hemirings via SU-h-ideals, *J. Intell. Fuzzy Systems*, **26**(3), (2014), 1363–1370.