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On Barycentric-Magic Graphs

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Abstract. Let A be an abelian group. A graph $G = (V, E)$ is said to be A-barycentric-magic if there exists a labeling $l : E(G) \longrightarrow A \setminus \{0\}$ such that the induced vertex set labeling $l^+ : V(G) \longrightarrow A$ defined by $l^+(v) = \sum_{uv \in E(G)} l(uv)$ is a constant map and also satisfies that $l^+(v) = \deg(v)l(u_v v)$ for all $v \in V$, and for some vertex u_v adjacent to v. In this paper we determine all $h \in \mathbb{N}$ for which a given graph G is $\mathbb{Z}_h$-barycentric-magic and characterize $\mathbb{Z}_h$-barycentric-magic labeling for some graphs containing vertices of degree 2 and 3.

Keywords: Magic graph, Barycentric sequences, Barycentric magic graph.

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1. Introduction

Let $G = (V, E)$ be a finite, simple and undirected graph and let A be an abelian group (written additively). The graph G is said to be A-*magic* if there exists a labeling $l : E(G) \longrightarrow A \setminus \{0\}$ of the edges of G by non-zero elements of A such that the induced vertex set labeling $l^+ : V(G) \longrightarrow A$ defined by $l^+(v) = \sum_{uv \in E(G)} l(uv)$ is a constant map. When this constant is 0, G is said to be A-*zero-sum magic*. If there exists a labeling l whose induced vertex set labeling is a constant map and for all $v \in V(G)$ the sum $l^+(v)$ also satisfies

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$l^+(v) = \deg(v)l(u_v v)$ for some vertex u_v adjacent to v, G is said to be *A-barycentric-magic*.

In this paper we study the following problem:

Given a graph G, find all $h \in \mathbb{N}$ for wich G is $\mathbb{Z}_h$-barycentric-magic and for those h, characterize $\mathbb{Z}_h$-barycentric-magic labeling for G.

The study of the barycentric-magic graphs is a new area in the labeled graph theory, specifically in magic labeling. It is motivated by the relationship between different types of magic labeling and the behavior of the sums of sequences in abelian groups. That is, A-magic labelings of a graph G are equivalent to sequences of non-zero elements of A with the same sum, A-zero-sum magic labelings are equivalent to zero-sum sequences in $A \setminus \{0\}$. In the same way, A-barycentric magic labelings are equivalent to sequences of non-zero elements of A that contain one element which is the "average" of its terms, called barycentric sequence and formally defined by:

Definition 1.1. Let $a_1, a_2, \dots a_k$ be k not necessarily distinct elements of an abelian group A. The above sequence is k-barycentric if there exists j such that $a_1 + a_2 + \cdots + a_j + \cdots + a_k = ka_j$. The element a_j is called a barycenter.

Barycentric sequences were introduced in [1, 2] as a natural extension of zero-sum sequences, and have already been used in graph labeling problems, specifically in Ramsey theory [1, 3, 5].

In this context we give the following definition:

Definition 1.2. Let A be an abelian group. A graph $G = (V, E)$ is said to be A-barycentric magic if there exists a labeling $l : E(G) \longrightarrow A \setminus \{0\}$ of the edges of G by non-zero elements of A such that the induced vertex set labeling $l^+ : V(G) \longrightarrow A$ satisfies:

(i) $l^+(v) = \sum_{uv \in E(G)} l(uv)$ is a constant map.

(ii) $l^+(v) = \deg(v)l(u_v v)$, for all $v \in V(G)$, and for some vertex u_v adjacent to v.

Notice that if A is a finite group of order n and $deg(v) \equiv 0 \pmod{n}$ for all $v \in V(G)$, the barycentric-magic graphs coincide with the zero-sum magic graphs.

Based on the definition of integer magic spectrum for a given graph G, which is the set of all positive integers h for which G is $\mathbb{Z}_h-$magic and is denoted by $IM(G)$, we define the barycenter-magic spectrum of G as follows:

Definition 1.3. For a given graph G the set of all positive integers h for which G is $\mathbb{Z}_h$-barycentric magic is called the barycenter-magic spectrum of G and is denoted by $BM(G)$.

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Besides this introduction, this paper contains two main sections. Section 2 presents the tools that are used in the next section. In Section 3 we give some general results about barycentric magic labeling for regular graphs and stars and solve the main problem for some graphs, particularly for graphs containing vertices of degree 2 and 3 such as chain of cycles, cycles with an edge or path joining two vertices, $K_{2,3}$ and $K_{2,n}$.

2. Tools

In this section we cite some results on barycentric sequences and magic graphs that will be used to establish the main results.

First, we have the following obvious lemma:

Lemma 2.1. *In every abelian group A, a sequence $a_1, a_2, \dots a_k$ where $a_1 = a_2 = \dots a_k = a \in A$ is k-barycentric.*

As we are interested in graphs containing vertices of degree 2 and 3, we use the following two lemmas that characterize barycentric sequences with lengths 2 and 3.

Lemma 2.2. *Let A be an abelian group. Any sequence in A with two elements is barycentric if and only if the elements are equal.*

Lemma 2.3. *A 3-sequence in $\mathbb{Z}_h$ is barycentric if and only if its elements are equal or are in arithmetic progression.*

In what follows, we present some results about magic labeling and the integer magic spectrum for a given graph G.

Lemma 2.4. *A graph G is $\mathbb{Z}_2$-magic if and only if the degree of every vertex of G is of the same parity.*

Lemma 2.5. *If G is a regular graph, then $IM(G) = \mathbb{N}$.*

Lemma 2.6. *$IM(P_2) = \mathbb{N}$ and for $n \geq 3$, $IM(P_n) = \varnothing$.*

In [4] Lee, Salehi and Sun showed the following theorem which determines the integer magic spectrum for stars.

Theorem 2.7. *Let $n \geqslant 3$, and ${p_1}^{\alpha_1}{p_2}^{\alpha_2}\cdots{p_k}^{\alpha_k}$ be the prime factorization of $n-1$. Then $IM(K_{1,n}) = \bigcup_{i=1}^{k} p_i\mathbb{N}$.*

3. Main Results

We start with some general results.

Lemma 3.1. *Every $\mathbb{Z}_2$-magic graph is a $\mathbb{Z}_2$-barycentric magic graph.*

Lemma 3.2. *For every abelian group A, P_2 is A-barycentric magic and P_n, $n \geq 3$, is not A-barycentric magic.*

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Let G be a regular graph. Then, according to Lemma 2.1, if we label all the edges of G with $a \in A \setminus \{0\}$, the graph G will be A-barycentric magic. In consequence we have:

Theorem 3.3. *If G is a regular graph, then $BM(G) = IM(G) = \mathbb{N}$.*

Particularly, when $G = K_{m,m}$ we have:

Corollary 3.4. $BM(K_{m,m}) = IM(K_{m,m}) = \mathbb{N}$.

For any $n \geqslant 1$ the complete bipartite graph $K_{1,n}$ is called a *star*. Note that $K_{1,1}$ is the same as P_2, which is A-barycentric magic for every abelian group A. Also $K_{1,2}$ is the same as P_3, which is not barycentric magic. For $n \geqslant 3$ we have the following result.

Theorem 3.5. *For any $n \geqslant 3$, if $K_{1,n}$ is $\mathbb{Z}_h$-magic, then it is $\mathbb{Z}_h$-barycentric magic.*

Proof. Let v_0 be the center of the star $K_{1,n}$, then $deg(v_0) = n$ and for the other vertices $v \neq v_0$, $deg(v) = 1$. If l is a magic labeling of $K_{1,n}$ all edges must have the same label, say $a \in \mathbb{Z}_h \setminus \{0\}$. Then $l^+(v_0) = na = deg(v_0)a$ and for $v \neq v_0$, $l^+(v) = a = deg(v)a$. Therefore, $K_{1,n}$ is $\mathbb{Z}_h$-barycentric magic. □

From Theorem 2.7 and Theorem 3.5 we obtain:

Corollary 3.6. *Let $n \geqslant 3$, and ${p_1}^{\alpha_1}{p_2}^{\alpha_2}\cdots{p_k}^{\alpha_k}$ be the prime factorization of $n-1$. Then $BM(K_{1,n}) = IM(K_{1,n}) = \bigcup_{i=1}^{k} p_i\mathbb{N}$.*

Now consider graphs containing vertices of degree 2 or 3. Let C_n be the cycle of length n. According to the Lemma 2.2, the only possible barycentric magic labeling of C_n is one in which all edges have the same label.

Theorem 3.7. *C_n is A-barycentric magic for every abelian group A and $BM(C_n) = IM(C_n) = \mathbb{N}$.*

The chain of cycles $C(n_1, n_2, \ldots, n_k)$ denotes the graph of k cycles C_{n_1}, C_{n_2},...,C_{n_k} of sizes $n_1, n_2, \ldots, n_k$ such that C_{n_i} and $C_{n_{i+1}}$ have a common vertex, for $i = 1, 2, \ldots, k-1$.

Theorem 3.8. *$C(n_1, n_2)$ is $\mathbb{Z}_h$-barycentric magic if and only if h is even.*

Proof. Let $G = C(n_1, n_2)$ be the graph of two cycles C_{n_1} and C_{n_2} with common vertex v. The degree set of G is $\{2, 4\}$, hence it is $\mathbb{Z}_2$-barycentric magic. From Lemma 2.2, all edges of each cycle should have the same label. Suppose that the edges of C_{n_1} are labeled with a and those of C_{n_2} with b. The requirement of having the same number for the sum of the edges incident to all vertices will provide the equations

$$2a \equiv 2b \pmod{h}$$

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for the vertices of the cycles, and

$$2a + 2b \equiv 2a \ (mod\ h) \quad or \quad 2a + 2b \equiv 2b \ (mod\ h)$$

for the vertex v.

From these equations we get:

$$2a \equiv 2b \equiv 0 \ (mod\ h).$$

This condition is fulfilled only when h is even, $h = 2m$ and $a = b = m$. Moreover, with this condition, the vertex v verifies that $l^+(v) = 2a + 2b = 0 = deg(v)a = deg(v)b$. Therefore $G = C(n_1, n_2)$ is $\mathbb{Z}_h$-barycentric magic. □

Corollary 3.9. $BM(C(n_1, n_2)) = 2\mathbb{N}$.

The previous result can be easily generalized to the case of k cycles.

Theorem 3.10. *$C(n_1, n_2, \ldots, n_k)$ is $\mathbb{Z}_h$-barycentric magic if and only if h is even.*

Corollary 3.11. $BM(C(n_1, n_2, \ldots, n_k)) = 2\mathbb{N}$.

Remark 3.12. The only $\mathbb{Z}_h$-barycentric magic labeling of $C(n_1, n_2, \ldots, n_k)$ is when h is even and all the edges are labeled with m, where $h = 2m$.

Theorem 3.13. *Let $G = C_n + e$, $n > 3$, where $e = uv$, u and v are non-adjacent vertices in C_n. Then G is $\mathbb{Z}_h$-barycentric magic if and only if $h = 4k$, $k \in \mathbb{N}$.*

Proof. Let $G = C_n + e$, $n > 3$ where $e = uv$, u and v are non-adjacent vertices in C_n. Then, the components of $G - \{u, v\}$ are the paths P_{n_1} and P_{n_2}. The degree set of G is $\{2, 3\}$, hence it is not $\mathbb{Z}_2$-barycentric magic. From Lemma 2.2, all the edges of P_{n_1} (and P_{n_2}) should have the same label. Moreover, from Lemma 2.3 the edges incident to u and v should have the same label or the labels must be in arithmetic progression.

Suppose that all edges have the same label a, then it must be that $3a \equiv 2a \pmod{h}$, this gives $a \equiv 0 \pmod{h}$. Hence the labeling of G will have three elements in arithmetic progression, one for P_{n_1}, another for P_{n_2} and the last one for the edge uv. This gives the three following cases:

Case 1. We label the graph G as follows. The edges of P_{n_1} with a, the edge uv with $a + r$ and the edges of P_{n_2} with $a + 2r$. The requirement of having the same number for the sum of the edges incident to all vertices will provide the equations

$$2a \equiv 2a + 4r \ (mod\ h),$$
$$3a + 3r \equiv 2a \ (mod\ h),$$
$$3a + 3r \equiv 2a + 4r \ (mod\ h).$$

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From these equations we get $a = r$, where $4r \equiv 0 \pmod{h}$. Then the barycentric magic labeling of G is as follows. The edges of P_{n_1} are labeled with a, those of P_{n_2} with $3a$ (or vice versa), and the edge uv with $2a$, where $4a \equiv 0 \pmod{h}$. Observe that if h is odd the condition $4a \equiv 0 \pmod{h}$ implies that $a \equiv 0 \pmod{h}$, which is not acceptable. And if $h = 4k + 2$, $k \in \mathbb{N}$, the condition $4a \equiv 0 \pmod{h}$ implies that $a = \frac{h}{2}$, hence $2a \equiv 0 \pmod{h}$ which is not an acceptable answer. Therefore the barycentric magic labeling is obtained only when $h = 4k$, $k \in \mathbb{N}$.

Case 2. We label the graph G as follows. The edges of P_{n_1} with $a + r$, the edge uv with a and the edges of P_{n_2} with $a + 2r$. If v_1 is a vertex of P_{n_1}, then the sum of the edges incident to v_1 and the sum of the edges incident to u must be the same. That is $2a + 2r \equiv 3a + 3r \pmod{h}$. Hence $a + r \equiv 0 \pmod{h}$, but $a + r$ is the label of the edges of P_{n_1}. Therefore, this labeling is not barycentric magic.

Case 3. We label the graph G as follows. The edges of P_{n_1} with $a + r$, the edge uv with $a + 2r$ and the edges of P_{n_2} with a. Using similar reasoning to the Case 2, we obtain again that $a + r \equiv 0 \pmod{h}$. Therefore, this labeling is not barycentric magic.

In consequence G is $\mathbb{Z}_h$-barycentric magic if and only if $h = 4k$, $k \in \mathbb{N}$. □

Corollary 3.14. *Let $G = C_n + e$, $n > 3$ where $e = uv$, u and v are non-adjacent vertices in C_n. Then $BM(G) = 4\mathbb{N}$.*

In the previous theorem, if we change the edge uv by a path, then the graph stops being barycentric magic.

Theorem 3.15. *Let G be the graph which consists of the cycle C_n with a path P_k connecting any two non-adjacent vertices u and v of C_n. Then G is not $\mathbb{Z}_h$-barycentric magic for any h.*

Proof. Let G be the graph C_n with a path P_k connecting any two non-adjacent vertices u and v of C_n. Let P_{n_1} and P_{n_2} be the two components of $G - P_k$. From Theorem 3.13, the only possible barycentric magic labeling of G is as follows. The edges of P_{n_1} are labeled with a, those of P_{n_2} with $3a$ (or vice versa), and the edges of P_k with $2a$. Then the vertex sum in P_k is $4a$, and the vertex sum in the others is $6a$. Hence we get $2a \equiv 0 \pmod{h}$, but $2a$ is the label of the edges of P_k. In consequence, G is not $\mathbb{Z}_h$-barycentric magic. □

Corollary 3.16. *Let G be the graph which consists of the cycle C_n with a path P_k connecting any two non-adjacent vertices u and v of C_n. Then $BM(G) = \emptyset$.*

For two cycles without common vertices joined by an edge or a path we have:

Theorem 3.17. *Let G be a graph which consists of two cycles C_{n_1} and C_{n_2} without common vertices joined by an edge uv with $u \in C_{n_1}$ and $v \in C_{n_2}$. Then G is not $\mathbb{Z}_h$-barycentric magic for any h.*

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Proof. Two edges incident to u are in C_{n_1}, then they must have the same label, say a. Hence, from Lemma 2.3 we have $l(uv) = a$. Therefore $3a \equiv 2a \pmod{h}$, that is $a \equiv 0 \pmod{h}$, which is not an acceptable answer. □

With the same argument of the previous theorem we obtain:

Theorem 3.18. *Let G be the graph which consists of two cycles C_{n_1} and C_{n_2} without common vertices joined by a path P_k. Then G is not $\mathbb{Z}_h$-barycentric magic.*

Now consider the complete bipartite graph $K_{2,3}$. In [6] Salehi showed that every magic labeling of $K_{2,3}$ induces zero-zum. In the case of barycentric magic labeling, we have:

Theorem 3.19. *$K_{2,3}$ is not $\mathbb{Z}_h$-barycentric magic for any h.*

Proof. Let $V(K_{2,3}) = \{u_1, u_2\} \cup \{v_1, v_2, v_3\}$ be the set of vertices of $K_{2,3}$. Then $deg(u_i) = 3$ for $i = 1, 2$, and $deg(v_j) = 2$ for $j = 1, 2, 3$. From Lemma 2.2 the edges incident to v_j must have the same label for each j, and from Lemma 2.3 the edges incident to u_i must have the same label or the labels are in arithmetic progression, for each i. We have the following two cases.
Case 1. We label the three edges incident to u_1 with the same label a. Then the three edges incident to u_2 must have the same label a (because the edges incident to v_j must have the same label). In this case, we have $3a \equiv 2a \pmod{h}$, hence $a \equiv 0 \pmod{h}$, which is not an acceptable answer.
Case 2. We label $K_{2,3}$ as follows. $l(u_1v_1) = a$, $l(u_1v_2) = a + r$ and $l(u_1v_3) = a + 2r$ Then, the edges incident to u_2 must be labeled in the same way, that is, $l(u_2v_j) = a + (j-1)r$ for $j = 1, 2, 3$ (because the edges incident to v_j must have the same label). From $l^+(v_1) = l^+(v_2)$ we have $2a \equiv 2a + 2r \pmod{h}$ or equivalently $2r \equiv 0 \pmod{h}$. And from $l^+(u_1) = l^+(v_1)$ we have $3a + 3r \equiv 2a \pmod{h}$, or $3a + r \equiv 2a \pmod{h}$, that is, $a + r \equiv 0 \pmod{h}$, which is not an acceptable answer.
In consequence $K_{2,3}$ is not $\mathbb{Z}_h$-barycentric magic for any h. □

Generalizing the previous theorem we have:

Theorem 3.20. *For $n \geq 3$, $K_{2,n}$ is $\mathbb{Z}_h$-barycentric magic if and only if $gcd(n-2, h) \neq 1$.*

Proof. Let $V(K_{2,n}) = \{u_1, u_2\} \cup \{v_1, v_2, \ldots, v_n\}$ be the set of vertices of $K_{2,n}$. Then $deg(u_i) = n$ for $i = 1, 2$, and $deg(v_j) = 2$ for $j = 1, 2, \ldots, n$. From Lemma 2.2, for each j, the edges incident to v_j must have the same label. Suppose that $l(u_1, v_j) = a_j$ and $l(u_2, v_j) = a_j$. Then $2a_1 \equiv 2a_2 \equiv \cdots \equiv 2a_n \pmod{h}$. We have the following two cases.
Case 1. h is odd. In this case, the condition $2a_1 \equiv 2a_2 \equiv \cdots \equiv 2a_n \pmod{h}$ implies that all edges have the same label, say a. From condition $l^+(u_i) =$

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$l^+(v_j)$ we obtain that $(n-2)a = 0 \pmod h$ and is it not possible when $gcd(n-2,h) = 1$. If $gcd(n-2,h) = \delta \neq 1$, then the choice $a = \dfrac{h}{\delta}$ gives a barycentric magic labeling.

Case 2. h is even. In this case, the condition $2a_1 \equiv 2a_2 \equiv \cdots \equiv 2a_n \pmod h$ implies that there are at most two different labels a and $b = a + \dfrac{h}{2}$, such that $2a \equiv 2b \pmod h$. We label $K_{2,n}$ as follows. $l(u_1, v_j) = a$ for $1 \leq j \leq k$ and $l(u_1, v_j) = b$ for $k+1 \leq j \leq n$, for some $1 \leq k \leq n$. Then, from Lemma 2.2 the edges incident to u_2 must be labeled in the same way. This labeling is barycentric magic if and only if:

$$ka + (n-k)b \equiv na \equiv 2a \equiv 2b \ \ (mod\ h) \tag{3.1}$$

or

$$ka + (n-k)b \equiv nb \equiv 2a \equiv 2b \ \ (mod\ h). \tag{3.2}$$

Without loss of generality, we consider only (3.1). The condition $na \equiv 2a \pmod h$ is satisfied only when $gcd(n-2,h) \neq 1$. Then suppose that $gcd(n-2,h) = \delta \neq 1$. Choose $k = \delta$, $a = \dfrac{h}{\delta}$, $b = a + \dfrac{h}{2}$ and using the fact that $n - \delta$ is always even we get

$$ka + (n-k)b = \delta\frac{h}{\delta} + (n-\delta)(\frac{h}{\delta} + \frac{h}{2}) \equiv na \ \ (mod\ h)$$

and

$$na \equiv 2b \equiv 2a \ \ (mod\ h).$$

Therefore $K_{2,n}$ is $\mathbb{Z}_h$-barycentric magic with this labeling. We can also label $K_{2,n}$ as in Case 1. □

The following example shows a particular case of the previous theorem.

Example 3.21. Consider the graph $K_{2,11}$ and the group $\mathbb{Z}_{12}$. Let $V(K_{2,11}) = \{u_1, u_2\} \cup \{v_1, v_2, \ldots, v_{11}\}$ be the set of vertices of $K_{2,11}$. Then $deg(u_i) = 11$ for $i = 1, 2$, and $deg(v_j) = 2$ for $j = 1, 2, \ldots, 11$. In this case choose $a = \dfrac{h}{\delta} = 4$ (because $gcd(n-2,h) = \delta = 3$), $b = a + \dfrac{h}{2} = 10$ and $k = 3$. The labeling of $K_{2,11}$ is as follows:

$$\begin{aligned} l(u_i v_j) &= 4 \qquad for\ \ i = 1,2\ \ and\ \ j = 1,2,3, \\ l(u_i v_j) &= 10 \quad for\ \ i = 1,2\ \ and\ \ j = 4, \ldots, 11. \end{aligned}$$

Then

$$\begin{aligned} l^+(v_j) &= 2 \cdot 4 = deg(v_j) \cdot 4 \equiv 8 \ (mod\, h)\ \ for\ \ j = 1,2,3, \\ l^+(v_j) &= 2 \cdot 10 = deg(v_j) \cdot 10 \equiv 8 \ (mod\, h)\ \ for\ \ j = 4, \ldots, 11, \\ l^+(u_i) &= 3 \cdot 4 + 8 \cdot 10 \equiv 8 \equiv deg(u_i) \cdot 4 \ (mod\, h)\ \ for\ \ i = 1,2. \end{aligned}$$

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Note that if we label all edges with $a = \frac{h}{\delta} = 4$ we also get a barycentric magic labeling.

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