Iranian Journal of Mathematical Sciences and Informatics
Vol. 18, No. 2 (2023), pp 107-115
DOI: 10.52547/ijmsi.18.2.107

Cordial Labeling of Corona Product between Paths and Fourth Power of Paths

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Abstract. A simple graph is said to be cordial if it admits 0-1 labeling that satisfies certain conditions. In this paper we investigate necessary and sufficient conditions for cordial labeling of the corona product between paths and fourth power of paths be cordial.

Keywords: Path, Corona, Cordial labeling, Fourth power.

2000 Mathematics subject classification: 05C78, 05C75, 05C20.

1. Introduction

Let G be a graph with p vertices and q edges. All graphs considered here are simple, finite, connected and undirected. A labeling of a graph G is a process of allocating numbers or labels to the nodes of G or lines of G or both through mathematical functions [2]. There are several types of labeling and a complete survey of graph labeling is available in [6]. Graph theory has a good development in the graph labeling and has a broad range of applications, some of which were reported in the work of Yegnanaryanan and Vaidhyanathan

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Received 20 April 2019; Accepted 26 March 2020

[9]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by Cahit in [3]. Extensions of this labeling include mean cordial labeling, H_1- and H_2-cordial labeling of some graphs [7]. In 1990, Cahit [4], proved the following: each tree is cordial; an Euerlian graph is not cordial if its size is congruent to $2(mod\ 4)$; a complete graph K_n is cordial if and only if $n \leq 3$ and a complete bipartite graph $K_{n,m}$ is cordial for all positive integers n and m. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona $G_1 \bigodot G_2$ of two graphs G_1 (with n_1 vertices , m_1 edges) and G_2 (with n_2 vertices , m_2 edges) is defined as the graph obtained by taking one copy of G_1 and copies of G_2 , and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 . It is easy to see that the corona $G_1 \bigodot G_2$ that has $n_1 + n_1 n_2$ vertices and $m_1 + n_1 m_2 + n_1 n_2$ edges. We will give a brief summary of definitions which are useful for the present investigations.

Definition 1.1. A mapping $f{:}V \to \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called *the label of the vertex v of G under f.* For any edge $e{=}uv$, the induced edge labeling $f^*{:}E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$, where $u, v \in V$. Let $v_f(i)$ be the numbers of vertices of G labeled i under f , and $e_f(i)$ be the numbers of edges of G labeled i under f^* where $i \in \{0, 1\}$.

Definition 1.2. A binary vertex labeling of a graph G is called *cordial* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if it admits a cordial labeling.

Definition 1.3. The fourth power of a cycles C_n denoted by C_n^4 , is $C_n \bigcup J$, where J is the set of all edges of the form edges $v_i v_j$ such that $2 \leq d(v_i v_j) \leq 4$, where $d(v_i v_j)$ is the shortest distance from v_i to v_j.

2. Terminology and notation

A path with m vertices and $m - 1$ edges, denoted by P_m, and its fourth power P_n^4 has n vertices and $4n - 10$ edges. We let L_{4r} denote the labeling 0011 0011...0011 "r-times", Let L'_{4r} denote the labeling 0110 0110...0110 "r-times". The labeling 1100 1100...1100 "r-times" and labeling 1001 1001...1001 "r-times" are written S_{4r} and S'_{4r}. Let M_{2r} denote the labeling 0101...01, zero-one "$r$times". We let M'_{2r} denote the labeling 1010...10. Regularly, we modify the labeling M_2r or M'_2r by adding symbols at one end or the other (or both). Also, L_{4r} (or L'_{4r}) with extra labeling from right or left (or both sides). Let us use α_i to indicate the labeling of P_n^4 that is adjacent to a vertex of P_m that is labeled i, $i = 0, 1$ of the corona $P_m \bigodot P_n^4$. Use y_i , b_i $(i = 0, 1)$ to denote the number of vertices and edges, respectively for α_0 of P_n^4, and consider y'_i, b'_i $(i = 0, 1)$ to denote the number of vertices and edges, respectively for α_1 of P_n^4. Sometimes, we use the notation $\alpha{*}_0$ for the labeling of P_n^4 which is only associated to the last vertex labeled 0 of P_m. In this case, we will use the

[DOI: 10.61186/ijmsi.18.2.107]

notation $b_0^{'*}$, $b_1^{'*}$, $y_0^{'*}$ and $y_1^{'*}$ instead of b'_0, b'_1, y'_0 and y'_1, respectively. Similarly, the notation $\alpha *_1$ may be used for the labeling of P_n^4 that is associated only to the last vertex labeled 1 of P_m. It is easy to verify that $v_0 = x_0 + x_0 y_0 + x_1 y'_0$, $v_1 = x_1 + x_0 y_1 + x_1 y'_1$, $e_0 = a_0 + x_0 b_0 + x_1 b'_0 + x_0 y_0 + x_1 y'_1$ and $e_1 = a_1 + x_0 b_1 + x_1 b'_1 + x_0(x_0 y_1) + x_1 y'_0$. Thus, $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y'_0 - y'_1)$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b'_0 - b'_1) + x_0(y_0 - y_1) - x_1(y'_0 - y'_1)$. When it comes to the proof, we only need to show that, for each specified combination of labeling, $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$.

3. Main results

In this section, we study cordial labeling of the corona product between paths and fourth power of paths and show that all $P_m \bigodot P_n^4$ are cordial for all integers $m \geq 1$ when $n \geq 7$, and for all integers $m > 1$ when $n = 3$.

Lemma 3.1. *The corona $P_m \bigodot P_3^4$ is cordial if and only if $m \neq 1$.*

Proof. Since $P_3^4 = C_3$, $P_m \bigodot P_3^4$ is cordial [8]. □

Lemma 3.2. *If $n \equiv 0 (mod\ 4)$, $n \geq 8$, then $P_m \bigodot P_n^4$ is cordial for all $m \geq 1$.*

Proof. Suppose that $n = 4s$, where $s \geq 2$. The following cases will be examined.

Case 1.

Suppose that $m = 1$. Choose the labeling 0 for P_1 and the labeling $\alpha_0 = 0L_{4s-4}01_2$ for P_{4s}^4. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_1 \bigodot P_{4s}^4$ is cordial. As an example, Figure (1) illustrates $P_1 \bigodot P_8^4$.

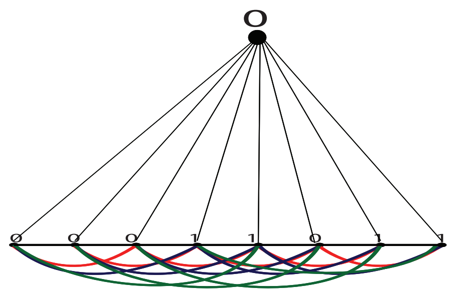

Figure 1

Case 2.

[DOI: 10.61186/ijmsi.18.2.107]

Suppose that $m = 2$. Choose the labeling 01 for P_2. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s - 5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s - 5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_2 \bigodot P^4_{4s}$ is cordial.

<u>Case 3</u>.

Suppose that $m = 3$. Choose the labeling 001 for P_3. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2, x_1 = 2, a_0 = a_1 = 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s - 5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s - 5$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_3 \bigodot P^4_{4s}$ is cordial.

<u>Case 4</u>. $m \equiv 0(mod\ 4)$.

Suppose that $m = 4r$, $r \geq 2$. Choose the labeling L_{4r} for P_{4r}. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 2r, a_0 = 2r, a_1 = 2r - 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s - 5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s - 5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \bigodot P^4_{4s}$ is cordial.

<u>Case 5</u>. $m \equiv 1(mod\ 4)$.

Suppose that $m = 4r + 1$, $r \geq 1$. Choose the labeling $L_{4r}0$ for P_{4r+1}. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2r + 1, x_1 = 2r, a_0 = a_1 = 2r, y_0 = y_1 = 2s, b_0 = b_1 = 8s - 5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s - 5$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+1} \bigodot P^4_{4s}$ is cordial.

<u>Case 6</u>. $m \equiv 2(mod\ 4)$.

Suppose that $m = 4r + 2$, $r \geq 1$. Choose the labeling $L_{4r}01$ for P_{4r+2}. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 2r + 1, a_0 = 2r, a_1 = 2r+1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+2} \bigodot P^4_{4s}$ is cordial.

<u>Case 7</u>. $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r + 3$, $r \geq 1$. Choose the labeling $L_{4r}001$ for P_{4r+3}. Take α_0 to be $0L_{4s-4}01_2$ and α_1 to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2r + 2, x_1 = 2r + 1, a_0 = a_1 = 2r + 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s - 5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0-v_1 = 1$ and $e_0-e_1 = 0$. Hence, $P_{4r+3} \bigodot P^4_{4s}$ is cordial. □

Lemma 3.3. *If $n \equiv 1(mod\ 4)$, then $P_m \bigodot P^4_n$ is cordial for all $m \geq 1$.*

Proof. Suppose that $n = 4s + 1$, where $s \geq 2$. The following cases will be examined.

<u>Case 1</u>.

Suppose that $m = 1$. Choose the labeling 0 for P_1 and the labeling $\alpha_0 = 1_2L'_{4s-4}010$ for P^4_{4s+1}. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = 2s, y_1 = 2s + 1, b_0 = b_1 = 8s - 3$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_1 \bigodot P^4_{4s+1}$ is cordial.

<u>Case 2</u>.

Suppose that $m = 2$. Choose the labeling 01 for P_2. Take α_0 to be $0_2L_{4s-4}101$ and α_1 to be $1_2L'_{4s-4}010$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 =$

[DOI: 10.61186/ijmsi.18.2.107]

$2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y_0' = 2s+1, y_1' = 2s$ and $b_0' = b_1' = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. As an example, Figure (2) illustrates $P_2 \bigodot P_9^4$. Hence, $P_2 \bigodot P_{4s+1}^4$ is cordial.

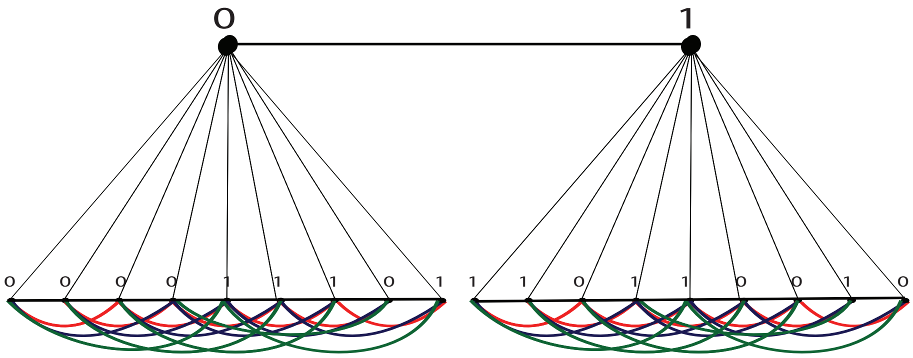

Figure 2

Case 3.
Suppose that $m = 3$. Choose the labeling 010 for P_3. Take α_0 (associated to the first vertex labeled 0 in P_3) to be $0_2L_{4s-4}101$, α_1 to be $1_2L_{4s-4}'010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in P_3) to be $1_2L_{4s-4}'010$. Therefore $x_0 = 2, x_1 = 1, a_0 = 0, a_1 = 2, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y_0' = 2s, y_1' = 2s+1, b_0' = b_1' = 8s-3, y_0^{'*} = 2s, y_1^{'*} = 2s+1$ and $b_0^{'*} = b_1^{'*} = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_3 \bigodot P_{4s+1}^4$ is cordial.

Case 4. $m \equiv 0 (mod\ 4)$.
Suppose that $m = 4r$, $r \geq 1$. Choose the labeling M_{4r} for P_{4r}. Take α_0 to be $0_2L_{4s-4}101$ and α_1 to be $1_2L_{4s-4}'010$. Therefore $x_0 = x_1 = 2r, a_0 = 0, a_1 = 4r-1, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y_0' = 2s, y_1' = 2s+1$ and $b_0' = b_1 = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \bigodot P_{4s+1}^4$ is cordial.

Case 5. $m \equiv 1 (mod\ 4)$.
Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $M_{4r}0$ for P_{4r+1}. Take α_0 to be $0_2L_{4s-4}101$, α_1 to be $1_2L_{4s-4}'010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in P_{4r+1}) to be $1_2L_{4s-4}'010$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = 0, a_1 = 4r, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y_0' = 2s, y_1' = 2s+1, b_0' = b_1' = 8s-3, y_0^{'*} = 2s, y_1^{'*} = 2s+1$ and $b_0^{'*} = b_1^{'*} = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+1} \bigodot P_{4s+1}^4$ is cordial.

Case 6. $m \equiv 2 (mod\ 4)$.
Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling M_{4r+2} for P_{4r+2}. Take α_0 to be $0_2L_{4s-4}101$ and α_1 to be $1_2L_{4s-4}'010$. Therefore $x_0 = x_1 = 2r+1, a_0 = 0, a_1 = 4r+1, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y_0' = 2s, y_1' = 2s+1$

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and $b'_0 = b'_1 = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence $P_{4r+2} \odot P^4_{4s+1}$ is cordial.

Case 7. $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $M_{4r+2}0$ for P_{4r+3}. Take α_0 to be $0_2L_{4s-4}101$, α_1 to be $1_2L'_{4s-4}010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in P_{4r+3} to be $1_2L'_{4s-4}010$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = 0, a_1 = 4r+2, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s, y'_1 = 2s+1, b'_0 = b'_1 = 8s-3, y'^{*}_0 = 2s, y'^{*}_1 = 2s+1$ and $b'^{*}_0 = b'^{*}_1 = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+3} \odot P^4_{4s+1}$ is cordial. □

Lemma 3.4. *If $n \equiv 2(mod\ 4)$, then $P_m \odot P^4_n$ is cordial for all $m \geq 1$.*

Proof. Suppose that $n = 4s+2$, where $s \geq 2$. The following cases will be studied.

Case 1.

Suppose that $m = 1$. Choose the labeling 0 for P_1 and the labeling $\alpha_0 = 01_30S_{4s-4}0$ for P^4_{4s+2}. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_1 \odot P^4_{4s+2}$ is cordial.

Case 2.

Suppose that $m = 2$. Choose the labeling 01 for P_2. Take α_0 to be $01_30S_{4s-4}0$ and α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_2 \odot P^4_{4s+2}$ is cordial.

Case 3.

Suppose that $m = 3$. Choose the labeling 001 for P_3. Take α_0 to be $01_30S_{4s-4}0$ and α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = 2, x_1 = 1, a_0 = a_1 = 1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. As an example, Figure (3) illustrates $P_3 \odot P^4_{10}$. Hence, $P_3 \odot P^4_{4s+2}$ is cordial.

Case 4. $m \equiv 0(mod\ 4)$.

Suppose that $m = 4r$, $r \geq 1$. Choose the labeling L_{4r} for P_{4r}. Take α_0 to be $01_30S_{4s-4}0$ and α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 2r, a_0 = 2r, a_1 = 2r-1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b_1 = 8s-1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \odot P^4_{4s+2}$ is cordial.

Case 5. $m \equiv 1(mod\ 4)$.

Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $L_{4r}0$ for P_{4r+1}. Take α_0 to be $01_30S_{4s-4}0$, α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = a_1 = 2r, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+1} \odot P^4_{4s+2}$ is cordial.

Case 6. $m \equiv 2(mod\ 4)$.

Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling $L_{4r}01$ for P_{4r+2} and

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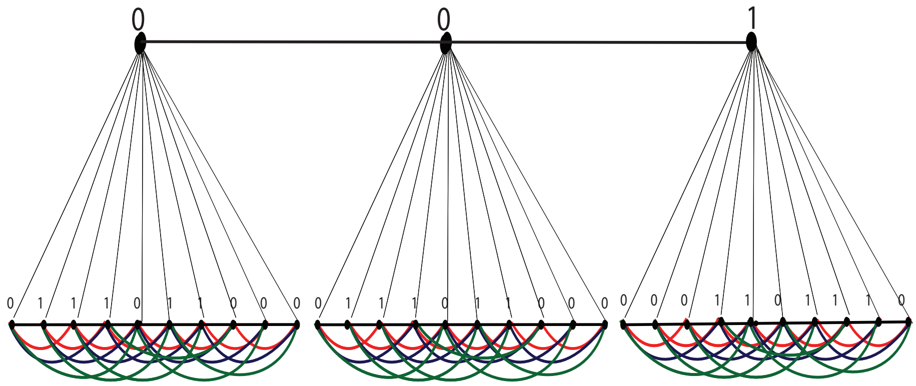

FIGURE 3

take α_0 to be $01_30S_{4s-4}0$ and α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 2r+1, a_0 = 2r, a_1 = 2r+1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+2} \bigodot P^4_{4s+2}$ is cordial.

<u>Case 7</u>. $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $L_{4r}001$ for P_{4r+3}. Take α_0 to be $01_30S_{4s-4}0$ and α_1 to be $0L_{4s-4}01_30$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = a_1 = 2r+1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+3} \bigodot P^4_{4s+2}$ is cordial. □

Lemma 3.5. *If $n \equiv 3(mod\ 4)$, then $P_m \bigodot P^4_n$ is cordial for all $m \geq 1$.*

Proof. Suppose that $n = 4s+3$, where $s \geq 1$. The following cases will be checked.

<u>Case 1</u>.

Suppose that $m = 1$. Choose the labeling 0 for P_1 and the labeling α_0= $1_2S_{4s}0$ for P^4_{4s+3}. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = 2s+1, y_1 = 2s+2, b_0 = b_1 = 8s+1$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_1 \bigodot P^4_{4s+3}$ is cordial.

<u>Case 2</u>.

Suppose that $m = 2$. Choose the labeling 01 for P_2. Take α_0 to be 0_21L_{4s} and α_1 to be $1_2S_{4s}0$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2$ and $b'_0 = b'_1 = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_2 \bigodot P^4_{4s+3}$ is cordial.

<u>Case 3</u>.

Suppose that $m = 3$. Choose the labeling 010 for P_3. Take α_0 to be 0_21L_{4s}, α_1 to be $1_2S_{4s}0$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in P_3) to be $1_2S_{4s}0$. Therefore $x_0 = 2, x_1 = 1, a_0 = 0, a_1 = 2, y_0 = 2s+2, y_1 = 2s+1, b_0 =$

$b_1 = 8s+1, y_0' = 2s+1, y_1' = 2s+2, b_0' = b_1' = 8s+1, y_0^{'*} = 2s+1, y_1^{'*} = 2s+2$ and$b_0^{'*} = b_1^{'*} = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_3 \odot P_{4s+3}^4$ is cordial.

Case 4. $m \equiv 0 (mod\ 4)$.

Suppose that $m = 4r$, $r \geq 1$. Choose the labeling M_{4r} for P_{4r}. Take α_0 to be $0_2 1 L_{4s}$ and α_1 to be $1_2 S_{4s} 0$. Therefore $x_0 = x_1 = 2r, a_0 = 0, a_1 = 4r-1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y_0' = 2s+1, y_1' = 2s+2$ and $b_0' = b_1' = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. As an example, Figure (4) illustrates $P_4 \odot P_7^4$. Hence, $P_{4r} \odot P_{4s+3}^4$ is cordial.

Case 5. $m \equiv 1 (mod\ 4)$.

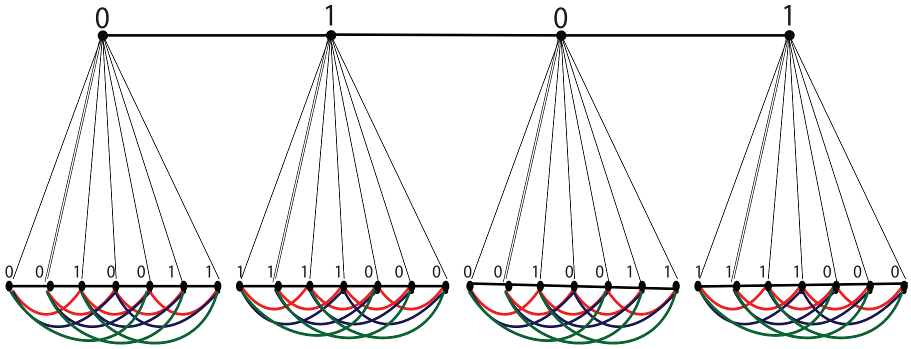

FIGURE 4

Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $M_{4r}0$ for P_{4r+1}. Take α_0 to be $0_2 1 L_{4s}$, α_1 to be $1_2 S_{4s} 0$ and $\alpha *_0$ (associated to the last vertex labeled 0 in P_{4r+1}) to be $1_2 S_{4s} 0$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = 0, a_1 = 4r, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y_0' = 2s+1, y_1' = 2s+2, b_0' = b_1' = 8s+1, y_0^{'*} = 2s+1, y_1^{'*} = 2s+2$ and $b_0^{'*} = b_1^{'*} = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+1} \odot P_{4s+3}^4$ is cordial.

Case 6. $m \equiv 2 (mod\ 4)$.

Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling M_{4r+2} for P_{4r+2}. Take α_0 to be $0_2 1 L_{4s}$ and α_1 to be $1_2 S_{4s} 0$. Therefore $x_0 = x_1 = 2r+1, a_0 = 0, a_1 = 4r+1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y_0' = 2s+1, y_1' = 2s+2$ and $b_0' = b_1' = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r+2} \odot P_{4s+3}^4$ is cordial.

Case 7. $m \equiv 3 (mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $M_{4r+2}0$ for P_{4r+3}. Take α_0 to be $0_2 1 L_{4s}$, α_1 to be $1_2 S_{4s} 0$ and $\alpha *_0$ (associated to the last vertex labeled 0 in P_{4r+3}) to be $1_2 S_{4s} 0$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = 0, a_1 = 4r+2, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y_0' = 2s+1, y_1' = 2s+2, b_0' =$

[DOI: 10.61186/ijmsi.18.2.107]

$b'_1 = 8s+1, y_0^{'*} = 2s+1, y_1^{'*} = 2s+2$ and $b_0^{'*} = b_1^{'*} = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+3} \bigodot P^4_{4s+3}$ is cordial.
As a consequence of all lemmas mentioned above we conclude that the corona product between paths and fourth power of paths is cordial for all $m, n \geq 7$. $\square$

Conclusion

In this paper we test the cordiality of the corona product between paths and fourth power of paths. We have shown that all $P_m \bigodot P^4_n$ are cordial for all integers $m \geq 1$ when $n \geq 7$, and for all integers $m > 1$ when $n = 3$.

Acknowledgments

The authors wish to thank the referees, for the careful reading of the paper and for the helpful suggestions and comments.

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[DOI: 10.61186/ijmsi.18.2.107]