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Wave Propagation in Generalized Thermodiffusion Elastic Medium with Impedence Boundary Condition

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Abstract. In the present investigation, we study the reflection of plane waves, that is, Longitudinal displacement wave(P-Wave), Thermal wave(T-Wave) and Mass Diffusive wave(MD-Wave) in thermodiffusion elastic-half medium which is subjected to impedence boundary condition in context of one relaxatioon time theory given by Lord and Shulman theory (L-S) and the Coupled theory (C-T) of thermoelasticity. The expressions of amplitude ratios are obtained numerically and their variation with angle of incidence is presented graphically for a particular model to emphasize on the impact of impedence parameter, relaxation time and diffusion. Some special cases are also deduced.

Keywords: Thermodiffusion, Amplitude Ratios, Plane waves, Impedence boundary.

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1. Introduction

Thermodiffusion is an active area of research since last decade due to its wide application in oil extraction field. Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration gradient, which is expressed as the change in the concentration due to change in position. The phenomenon of coupling of the fields of temperature, mass diffusion and that of strain in an elastic solid causes the thermodiffusion. Nowacki [1, 2, 3, 4] in series of paper introduced the concept of thermodiffusion by using the coupled thermoelastic model.

The phenomenon of reflection of P and SV waves from free surface of an elastic soild in generalized thermodiffusion elastic medium is studied by Singh [5]. Singh [6] also discussed the reflection phenomena of SV-wave from free surface of an elastic solid with generalized thermoelastic diffusion. Aouadi [7] studied the uniqueness and reciprocity theorems in generalized theory of thermoelastic diffusion medium.

Aouadi [8] invesitgated a problem for an infinite elastic body with spherical cavity in generalized thermoelastic diffusion medium. Sharma et.al. [9] studied the propagation of surface waves in heat conducting material in elasto-thermodiffusive medium. Abo Dahab and Singh [10] studied the influence of magnetic field in an elastic solid half-space under thermoelastic diffusion. Kumar and Panchal [11] investigated the problem of propagation of axial symmetric cylindrical surface waves in a cylindrical bore through a homogeneous isotropic thermoelastic diffusion medium in the context of theories of generalized thermoelasticity. Kumar and Kansal [12] studied the propagation of plane waves and also obtained fundamental solutions in the theory of thermoelastic diffusion.

Impedence boundary conditions are defined as linear combination of unknown functions and their derivtives described on the boundary. It is commonly used in the fields of acoustics, electromagnetism and in the area of seismology. Tiersten [13] discussed the effect of thin layer of different materials over an elastic half space with the help of impedence boundary conditions. The propagation of Rayleigh waves is investigated by Malischewsky [14] using impedence boundary conditions. Vinh and Hue [15] discussed propagation of Rayleigh waves in an incompressible elastic half-space with impedence boundary conditions and derived explicit secular equations of the wave. Singh [16] discussed the problem on the reflection of elastic waves at a plane surface of an elastic half-space subjected to impedance boundary conditions. Singh et al. [17] investigated the coupled partial differential equations governing a rotating thermoelastic

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medium in context of Lord and Shulman theory and are solved for plane wave solutions which is subjected to impedance boundary conditions.

As the propagation of thermoelastic waves in such a medium have been continued interest due to its importance in various fields such as oil extraction, soil dynamics and geophysics. Keeping in view of this, the present paper is devoted to the study of propagation of plane waves at free space in thermodiffusion elastic medium to determine the impact of relaxation times, diffusion and impedence parameter on amplitude ratio of reflected waves and also some particular cases are deduced.

2. Basic equations

Following Sherief et.al [18], the governing equations for isotropic homogeneous heat conductive diffusion elastic half-space in absence of body forces, heat sources and diffusive mass sources are:
The constitutive relation is defined as:

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{k,k} - \beta_1 T - \beta_2 C], \tag{2.1}$$

Stress equation of motion:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} = \rho \ddot{u}_i, \tag{2.2}$$

Equation of heat conduction:

$$\rho C_E(1 + \tau_0 \frac{\partial}{\partial t})\frac{\partial T}{\partial t} + \beta_1 T_0(1 + \tau_0 \frac{\partial}{\partial t})\frac{\partial e}{\partial t} + aT_0(1 + \tau_0 \frac{\partial}{\partial t})\frac{\partial C}{\partial t} = K^* T_{,ii}, \tag{2.3}$$

Equation of mass diffusion:

$$D\beta_2 e_{,ii} + DaT_{,ii} + (1 + \tau^0 \frac{\partial}{\partial t})\frac{\partial C}{\partial t} - DbC_{,ii} = 0. \tag{2.4}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (i, j = 1, 2, 3)$$
$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c,$$

λ, μ- Lame's constants, α_t- coefficient of linear thermal expansion, α_c - coefficient of diffusion expansion, ρ- density , K^*- thermal conductivity, t_{ij} - components of stress tensor,e_{ij}- components of strain tensor, $e = e_{kk}$, δ_{ij}-kronecker delta, u_i- displacement component, C-concentration, C_E-specific heat at constant strain, a,b- constants, t-time, T-absolute temperature, T_0-temperature of medium in its natural state assumed to be such that $|\frac{T}{T_0}| < 1$, D-thermoelastic diffusion constant, τ_0- thermal relaxation time, τ^0-diffusion relaxation time.

For L-S Theory $\quad \tau_0 \geq \tau^0 \geq 0.$
For C-T Theory $\quad \tau_0 = \tau^0 = 0.$

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3. Formulation and solution of the problem

We have taken a homogeneous, isotropic with diffusion in elastic half-space. The rectangular cartesian co-ordinate system (x_1, x_2, x_3) having origin at the interface x_3=0 is considered along with x_3-axis pointing normally into the medium as shown in figure 1. Plane waves in x_1x_3-plane with wave front parallel to x_2-axis, therefore all the field variables depend only on x_1, x_3, and t.

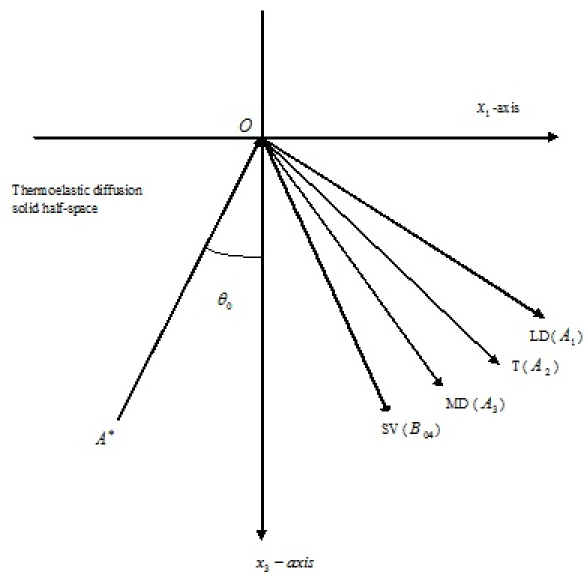

Figure 1. Geometry of the problem.

The problem considered here is two dimensional, therefore we take

$$\overrightarrow{u} = (u_1, 0, u_3). \tag{3.1}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$(x_1^{'}, x_3^{'}) = \frac{\omega_1}{c_1}(x_1, x_3)\,, \quad (u_1^{'}, u_3^{'}) = \frac{\rho c_1 \omega_1}{\beta_1 T_0}(u_1, u_3)\,, \quad (t_{33}^{'}, t_{31}^{'}) = \frac{1}{\beta_1 T_0}(t_{33}, t_{31})\,,$$

$$C^{'} = \frac{\beta_2}{\rho c_1^2} C\,, \quad T^{'} = \frac{\beta_1}{\rho c_1^2} T\,, \quad (\tau_0^{'}, \tau^{0'}, t^{'}) = \omega_1(\tau_0, \tau^0, t)\,, \tag{3.2}$$

where

$$c_1^2 = \left(\frac{\lambda + 2\mu}{\rho}\right) \quad and \quad \omega_1 = \frac{\rho C_E c_1^2}{K^*}.$$

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The expression relating displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ to the scalar potential functions $q(x_1, x_3, s)$ and $\psi(x_1, x_3, s)$ in dimensionless form are given by

$$(u_1, u_3) = \frac{\partial}{\partial x_1}(q, \psi) + \frac{\partial}{\partial x_3}(-\psi, q). \tag{3.3}$$

Making use of equations (3.1)-(3.2) in equations (2.2), (2.3) and (2.4) (suppressing the primes for convenience) and assuming the motion to be harmonic, we can write

$$(q, \psi, T, C) = (\tilde{q}, \tilde{\psi}, \tilde{T}, \tilde{C})e^{\iota\omega t}$$

(omitting the bars), we obtain the following set of equations as:

$$\begin{aligned}
&[(1 + a_1)\nabla^2 + \omega^2]q - a_2T - a_2C = 0,\\
a_3[(1 + \tau_0\iota\omega)\iota\omega\nabla^2]q + [(1 + \tau_0\iota\omega)\iota\omega &- \nabla^2]T + a_4[(1 + \tau_0\iota\omega)\iota\omega]C = 0,\\
a_5\nabla^4 q + a_6\nabla^2 T + [a_7(1 + &\tau^0\iota\omega)\iota\omega - a_8\nabla^2]C = 0,\\
&[a_1\nabla^2 + \omega^2]\psi = 0,
\end{aligned} \tag{3.4}$$

where ω is the angular frequency and

$$a_1 = \frac{\mu}{\rho c_1^2}, \quad a_2 = \frac{\rho c_1^2}{\beta_1 T_0}, \quad a_3 = \frac{\beta_1^3 T_0^2}{\rho^2 c_1^2 \omega_1 K^*}, \quad a_4 = \frac{a\beta_1 T_0 c_1^2}{\beta_2 \omega_1 K^*},$$

$$a_5 = \frac{D\beta_1\beta_2 T_0\omega_1^2}{\rho c_1^4}, \quad a_6 = \frac{Da\rho\omega_1^2}{\beta_1}, \quad a_7 = \frac{\rho\omega_1 c_1^2}{\beta_2}, \quad a_8 = \frac{Db\rho\omega_1^2}{\beta_2}.$$

Any one of the waves from (Longitudinal displacement wave, Thermal wave, Mass diffusive wave and Shear wave) considered to be incident at the free surface $x_3 = 0$ making inclination θ_0 to the normal of the surface. For each incident wave, we get reflected Longitudinal displacement wave (P-Wave), Thermal wave (T-Wave), Mass Diffusive wave (MD-Wave) and Shear wave (SV-Wave) as shown in figure 1.

For the purpose of solving the equations (3.4), we assume the solution of the form

$$(q, \psi, T, C) = (q_0, \psi_0, T_0, C_0)e^{\iota\kappa(x_1 sin\theta - x_3 cos\theta + \upsilon t)},$$

where κ stands for the wave number, ι is known as iota, θ is the angle of inclination and quantities such as q_0, ψ_0, T_0, C_0 are arbitrary constants. Using the values of (q, ψ, T, C) given by above equation in (3.4) yields.

Making use of above equation in equations (3.4), yields

$$\upsilon^6 - F\upsilon^4 + G\upsilon^2 - H = 0, \tag{3.5}$$

$$\upsilon^2 - a_1 = 0, \tag{3.6}$$

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where $\upsilon(=\frac{\omega}{\kappa})$ is the velocity of coupled waves; υ_1, υ_2, υ_3 are the velocities of the coupled waves namely P-wave, T-wave, MD-wave and υ_4 is the velocity of the SV-wave and

$$F=\frac{a_7[1+a_1+a_2a_3]\tau_g\tau_h+[a_8-a_4a_6]\tau_g+a_7\tau_h}{a_7\tau_g\tau_h},\quad \tau_g=\tau_0-\frac{\iota}{\omega},$$

$$G=\frac{(1+a_1)[a_8-a_4a_6]\tau_g+(1+a_1)a_7\tau_h+a_8-G_1}{a_7\tau_g\tau_h},\quad \tau_h=\tau^0-\frac{\iota}{\omega},$$

$$G_1=a_2[a_4a_5+a_3a_8+a_3a_6+a_5]\tau_g,\quad H=\frac{(1+a_1)a_8-a_2a_5}{a_7\tau_g\tau_h}.$$

4. Boundary conditions

The required boundary conditions at the free surface $x_3=0$ are

$$(i)\quad t_{33}+\omega Z_1u_3=0,$$
$$(ii)\quad t_{31}+\omega Z_2u_1=0,$$
$$(iii)\quad \frac{\partial T}{\partial x_3}=0,$$
$$(iv)\quad \frac{\partial C}{\partial x_3}=0, \tag{4.1}$$

where Z_1 and Z_2 are proportional coefficients called as Impedence parameters. The traction free boundary conditions are recovered by setting $Z_1=0$ and $Z_2=0$.

We assume that the values of $q,\ T,\ C$ and ψ satisfying the boundary conditions (4.1) as

$$[q,T,C]=\Sigma(1,d_l,f_l)(A_{0l}e^{\iota\kappa_l(x_1sin\theta_0-x_3cos\theta_0)+\iota\omega t}+A_ie^{\iota\kappa_l(x_1sin\theta_l+x_3cos\theta_l)+\iota\omega t}), \tag{4.2}$$

$$\psi=(B_{04}e^{\iota\kappa_4(x_1sin\theta_0-x_3cos\theta_0)+\iota\omega t}+B_4e^{\iota\kappa_4(x_1sin\theta_4+x_3cos\theta_4)+\iota\omega t}), \tag{4.3}$$

where

$$d_l=\frac{[a_8(1+a_1)-a_2a_5]\kappa_l^4-[a_8+a_7(1+a_1)\tau_h]\omega^2\kappa_l^2+a_7\omega^4\tau_h}{a_2a_7\tau_h\omega^2+a_2[a_6+a_8]\omega^2\kappa_l^2},$$

$$f_l=\frac{[a_2a_5+a_6(1+a_1)]\kappa_l^4-a_6\omega^2\kappa_l^2}{a_2a_7\tau_h\omega^2+a_2[a_6+a_8]\omega^2\kappa_l^2},\qquad (l=1,2,3)$$

where A_{0l} are the amplitude of the incident P-wave, T-wave, MD-wave and B_{04} is the amplitude of the incident SV-wave. A_l are the amplitude of the reflected P-wave, T-wave, MD-wave and B_4 is the amplitude of the reflected SV-wave.

Snell's Law is given as

$$\frac{\sin\theta_0}{\upsilon_0}=\frac{\sin\theta_1}{\upsilon_1}=\frac{\sin\theta_2}{\upsilon_2}=\frac{\sin\theta_3}{\upsilon_3}=\frac{\sin\theta_4}{\upsilon_4}, \tag{4.4}$$

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where

$$\kappa_1\upsilon_1=\kappa_2\upsilon_2=\kappa_3\upsilon_3=\kappa_4\upsilon_4=\omega, at\ x_3=0 \tag{4.5}$$

$$\upsilon_0=\begin{cases}\upsilon_1, & \text{for incident} \quad \text{LD}-\text{wave}\\ \upsilon_2, & \text{for incident} \quad \text{T}-\text{wave}\\ \upsilon_3, & \text{for incident} \quad \text{MD}-\text{wave}\\ \upsilon_4, & \text{for incident} \quad \text{SV}-\text{wave}\end{cases}$$

Considering the phase of the reflected waves can easily write using the equations (4.4)-(4.5)

$$\frac{\cos\theta_j}{\upsilon_j}=\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}}. \tag{4.6}$$

Following Schoenberg [19], if we write

$$\frac{\cos\theta_j}{\upsilon_j}=\frac{\cos\theta_j^{'}}{\upsilon_j^{'}}+\iota\frac{c_j}{2\pi\upsilon_0}, \qquad (j=1,2,3),$$

$$\frac{\cos\theta_j^{'}}{\upsilon_j^{'}}=\frac{1}{\upsilon_0}R_e\left\{\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}}\right\}, \quad c_j=2\pi I_m\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}},$$

where $\upsilon_j^{'}$, the real phase speed and $\theta_j^{'}$, the angle of reflection are given by

$$\frac{\upsilon_j^{'}}{\upsilon_0}=\frac{\sin\theta_j^{'}}{\sin\theta_0}\left[\sin^2\theta_0+\left[R_e\left(\left[(\upsilon_0/\upsilon_4)^2-\sin^2\theta_0\right]^{\frac{1}{2}}\right)\right]^2\right]^{\frac{-1}{2}},$$

and c_j, the attenuation in a depth is equal to the wavelength of incident waves i.e. $(2\pi\upsilon_0)/\omega$.

Making use of the potential given by equations (3.3) in the boundary conditions (4.1) and with the help of equations (4.2)-(4.3), we get a system of homogenous equations which can be written as

$$\sum a_{ij}R_j=Y_j,(j=1,2,3,4),$$

where

$$a_{1j}=-2a_1\kappa_j^2\left(\frac{\upsilon_j}{\upsilon_0}\right)\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}}-a_9\kappa_j^2-a_2d_j-A_j,$$

$$a_{14}=-2a_1\kappa_4^2\left(\frac{\upsilon_4}{\upsilon_0}\right)^2\left[\left(\frac{\upsilon_0}{\upsilon_4}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}}+\iota\omega Z_1\kappa_4\left(\frac{\upsilon_4}{\upsilon_0}\right)^2\sin\theta_0,$$

$$a_{2j}=-2a_1\kappa_j^2\left(\frac{\upsilon_j}{\upsilon_0}\right)\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2-\sin^2\theta_0\right]^{\frac{1}{2}}+\iota\omega Z_2\kappa_j\left(\frac{\upsilon_j}{\upsilon_0}\right)\sin\theta_0,$$

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$$a_{24} = -a_1\kappa_4^2\left(\frac{\upsilon_4}{\upsilon_0}\right)^2\sin^2\theta_0 - \iota\omega Z_2\kappa_4\left(\frac{\upsilon_4}{\upsilon_0}\right)\left[\left(\frac{\upsilon_0}{\upsilon_4}\right)^2 - \sin^2\theta_0\right]^{\frac{1}{2}},$$

$$a_{3j} = d_j\iota\kappa_j\left(\frac{\upsilon_j}{\upsilon_0}\right)\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2 - \sin^2\theta_0\right]^{\frac{1}{2}}, \qquad a_{34} = 0,$$

$$a_{4j} = f_j\iota\kappa_j\left(\frac{\upsilon_j}{\upsilon_0}\right)\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2 - \sin^2\theta_0\right]^{\frac{1}{2}}, \qquad a_{44} = 0,$$

$$A_j = a_2f_j + \iota\omega Z_1\kappa_j\left[\left(\frac{\upsilon_0}{\upsilon_j}\right)^2 - \sin^2\theta_0\right]^{\frac{1}{2}}.$$

5. Particular Cases

5.1 Neglecting Diffusion Effect(a=b=D=0):

We obtain the corresponding expression for thermoelastic half space are given as:

$$\begin{aligned}
[(1+a_1)\nabla^2+\omega^2]q - a_2T &= 0,\\
a_3[(1+\tau_0\iota\omega)\iota\omega\nabla^2]q + [(1+\tau_0\iota\omega)\iota\omega - \nabla^2]T &= 0,\\
[a_1\nabla^2+\omega^2]\psi &= 0
\end{aligned}$$

5.2 Neglecting Relaxation Times(τ_0=τ^0=0):

We obtain the corrsponding expressions for thermodiffusion elastichalf space are given as:

$$\begin{aligned}
[(1+a_1)\nabla^2+\omega^2]q - a_2T - a_2C &= 0,\\
a_3\iota\omega\nabla^2q + [\iota\omega-\nabla^2]T + a_4\iota\omega C &= 0,\\
a_5\nabla^4q + a_6\nabla^2T + [a_7\iota\omega - a_8\nabla^2]C &= 0,\\
[a_1\nabla^2+\omega^2]\psi &= 0
\end{aligned}$$

6. Numerical Results and Discussion

In order to illustrate theoretical results pbtained in the proceeding section, we now present some numerical results.

Following Sherief and Saleh [20] copper material is choosen:

$$\lambda = 7.76\times10^{10}Kgm^{-1}s^{-2}, \quad \mu = 3.86\times10^{10}Kggm^{-1}s^{-2}, \quad \rho = 8954Kgm^{-3},$$

$$K* = 1.0\times10^{10}Wm^{-1}K^{-1}, \quad T_0 = 293K, \quad C_E = 383.1JKg^{-1}K*^{-1}$$

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Following Thomas [21], diffusion parameter are:

$$\alpha_t = 1.78 \times 10^{-5} K^{-1}, \quad \alpha_c = 1.98 \times 10^{-4} m^3 Kg^{-1}, \quad a = 1.2 \times 10^4 m^2 s^{-2} K^{-1},$$

$$b = 0.9 \times 10^6 m^5 Kg^{-1} s^{-2}, \quad D = 0.85 \times 10^{-8} Kgm^{-3},$$

The thermal relaxation times and the diffusion relaxation times are taken as:

$$\tau_0 = 0.2s, \quad \tau^0 = 0.1s.$$

The variations of amplitude ratios for plane waves in thermodiffusion elastic half-space along with impedance conditions are shown graphically in Figures 2-13 with angle of incidence $0^0 \leq \theta_0 \leq 90^0$ for incident P-wave, T-wave and MD-wave. The solid line and dashed line corresponds to the case of coupled theory of thermoelasticity for impedance parameter (Z_1=10 and Z_2=10) and non-impedance parameter (Z_1=0 and Z_2=0) and whereas the solid line with center symbol 'diamond' and the dashed line with center symbol 'triangle' represents the case of L-S theory of thermoelasticity with impedance parameter (Z_1=10 and Z_2=10) and non-impedance parameter (Z_1=0 and Z_2=0).

6.1 Incident P-Wave:

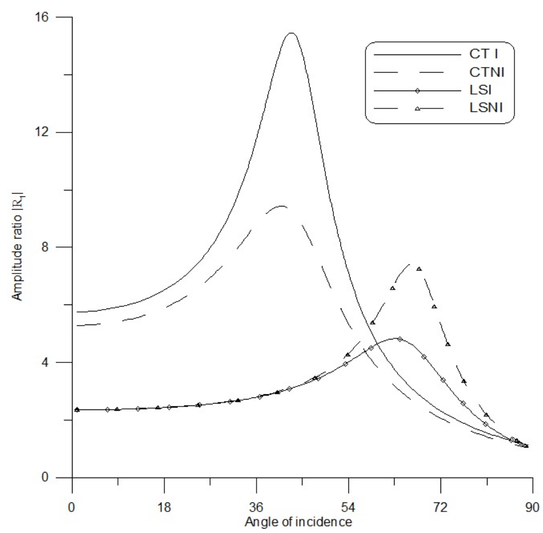

FIGURE 2

Fig.2 depicts the variation of amplitude ratio $|R_1|$ along with the angle of incidence θ_0. It is observed that in first half of range the values of amplitude ratio $|R_1|$ in case of CT-Theory are greater in magnitude as compared to those observed for LS-Theory whereas trends are reversed in later half.

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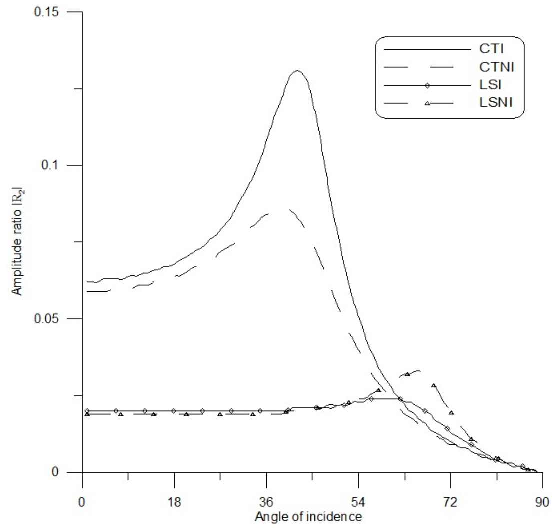

FIGURE 3

Fig.3 shows that the amplitude ratio $|R_2|$ for LSI and LSNI shows steady state in the range $0^0 \leq \theta_0 \leq 54^0$, as θ_0 increases values tends to approach zero value. It is also noticed that values of CTI and CTNI increases in the range $0^0 \leq \theta_0 \leq 45^0$ with greater magnitude as compared to those observed for L-S theory and as θ_0 increases, values tends to approaches to origin with decreasing magnitude, revealing the impact of relaxation time.

Fig.4 is a plot of amplitude ratio $|R_3|$ along with the angle of incidence θ_0. It is observed that a trend of variation of amplitude ratio $|R_3|$ is similar to those observed for $|R_2|$ with significant difference in their values.

Fig.5 depicts that the variation of amplitude ratio $|R_4|$. It is noticed that there is decrease in trends of curves for both theories of thermoelastic in case of non-impedance boundary while opposite trends are noticed for impedence parameter. It is also observed that values of amplitude ratio for CTNI are greater as compared to those noticed for LSNI.

6.2 Incident T-Wave:

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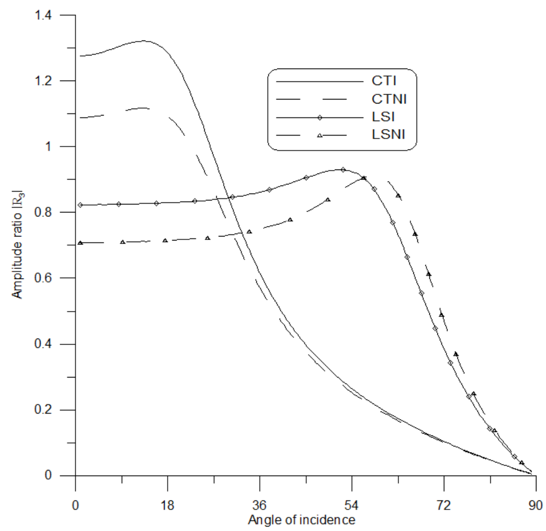

FIGURE 4

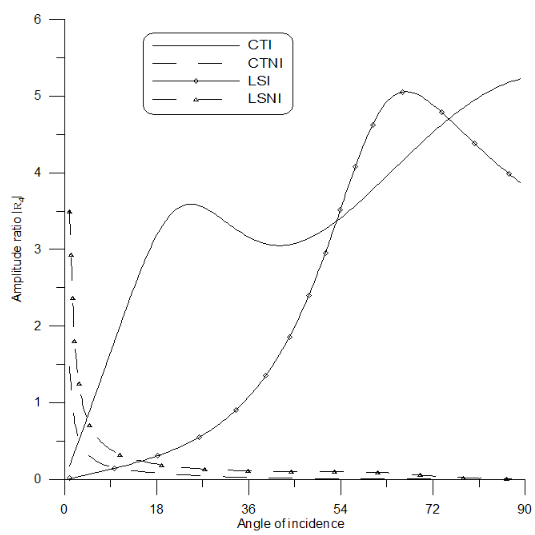

FIGURE 5

Fig.6 depicts that the trend of values of amplitude ratio $|R_1|$ for CT-Theory with impedance parameter and non-impedance parameter are similar in nature in the entire range with significant difference in their magnitude and ultimately the values approaches to zero. Also LSI and LSNI shows steady behaviour in the range $0^0 \leq \theta_0 \leq 63^0$ and with the increase in θ_0, values of $|R_1|$ decreases.

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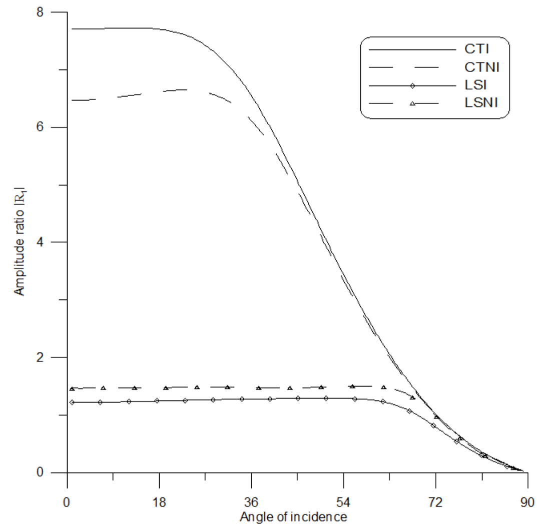

FIGURE 6

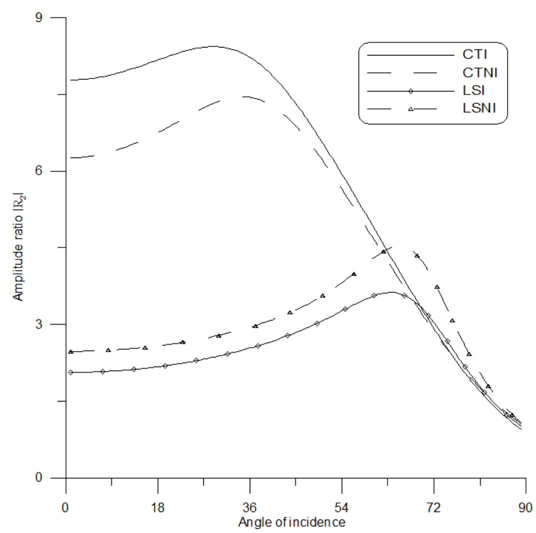

FIGURE 7

Fig.7 is a plot of amplitude ratio $|R_2|$ with the angle of incidence θ_0. It is observed that trends for both cases of thermoelasticity are similar in nature. It is also observed that for both cases of impedence parameter the values of $|R_2|$ for CT-Theory, are greater as compared to those observed for LS-Theory in $0^0 \leq \theta_0 \leq 65^0$ and vice-versa trends are observed with increase in θ_0.

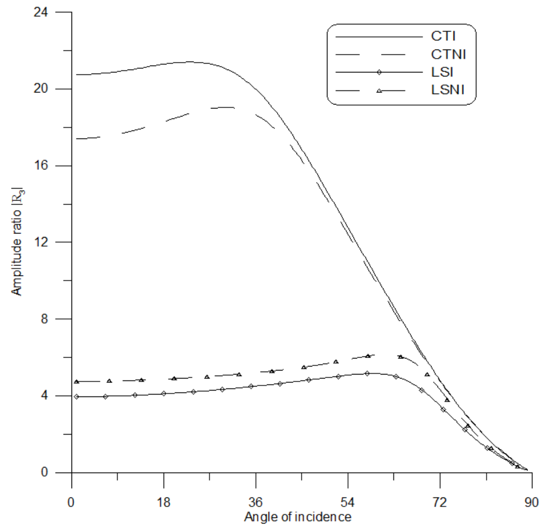

FIGURE 8

Fig.8 shows the considerable effect of impedence parameter on the amplitude ratio $|R_3|$ for CT-Theory. Also CTI and CTNI shows similar kind of variations in entire range. For L-S theory values of $|R_3|$ in absence of impedence parameter (LSNI) are greater than those observed for LSI in entire range and with increase in θ_0 values tend towards origin.

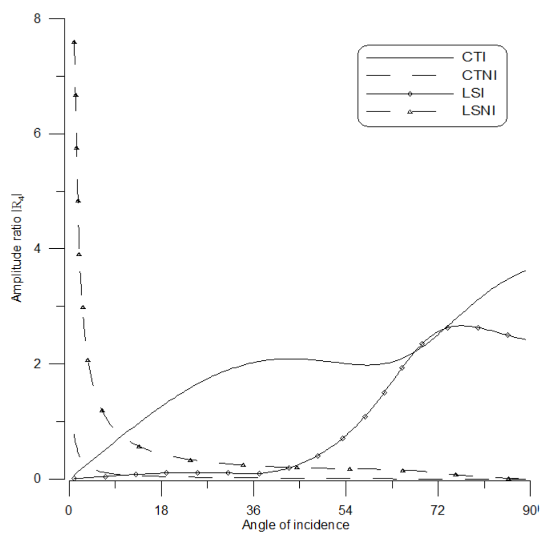

FIGURE 9

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The variation of amplitude ratio $|R_4|$ with the angle of incidence θ_0 is shown in Fig.9. It is noticed that initially the amplitude ratio for CTNI and LSNI decreases sharply and later on shows small variations about zero value. Also the trends of curves for CTI in case of impedance parameter shows an opposite behaviour to those observed for LSI, reveals the impact of relaxation time.

6.3 Incident MD-Wave:

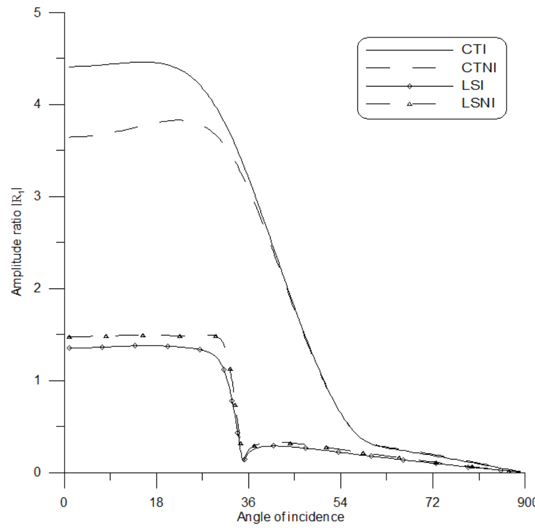

Figure 10

Fig.10 is a plot of amplitude ratio $|R_1|$ along with the angle of incidence θ_0. It is noticed that the values for LSI and LSNI shows steady state about 1.5 in the range $0^0 \leq \theta_0 \leq 27^0$ and decreases towards origin in the remaining range. Also the values of $|R_1|$ for CTI and CTNI shows staedy state initially and for the left over interval it approaches towards the origin.

Fig.11 depicts the variations of $|R_2|$ for CT-theory and L-S theory. The impact of relaxation time is noticable for both values of impedence parameters. It is observed that values of $|R_2|$ for L-S theory increases sharply in the range $0^0 \leq \theta_0 \leq 30^0$, whereas values of $|R_2|$ for CT theory shows staedy state and with increase in θ_0 values of $|R_2|$ approaches to zero value.

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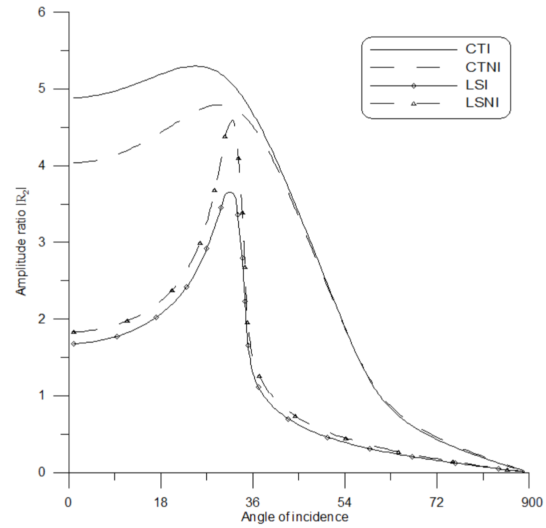

FIGURE 11

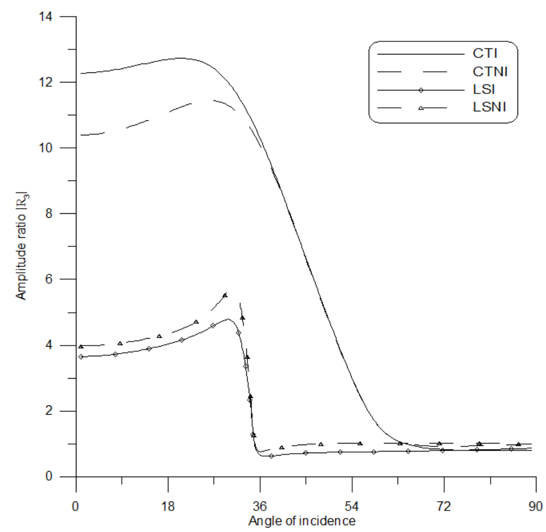

FIGURE 12

Fig.12 is the variation of amplitude ratio of $|R_3|$ along with the angle of incidence. It is observed that initially the values for L-S theory and CT-theory with impedance and non-impedance parameter increases in the range $0^0 \leq \theta_0 \leq 27^0$ and with further increase in θ_0 the values of $|R_3|$ for L-S theory shows steady state about 1.5 whereas the values for CT theory decreases in entire range, which reveals the impact of relaxation time.

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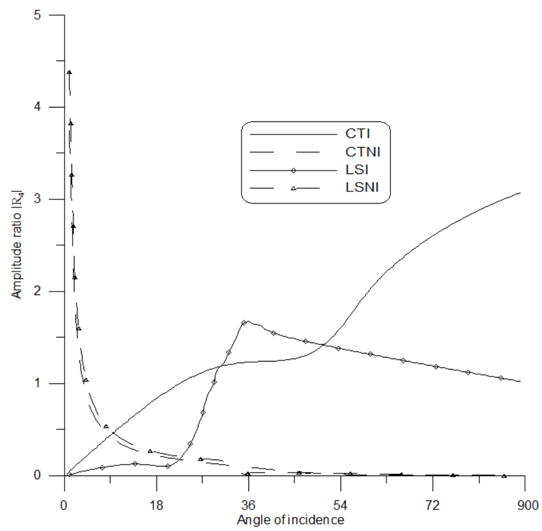

FIGURE 13

Fig.13 shows the variation of amplitude ratio $|R_4|$. the impact of impedence parameter is clearly noticable on both theories of thermoelasticity. It is noticed that LSI and CTI with $Z_1 = 10$ and $Z_2 = 10$ shows an opposite behaviour to each other in the whole range whereas there is an overall decrease in trends of curves for LSNI and CTNI with $Z_1 = 0$ and $Z_2 = 0$, revealing the impact of impedance paramter on amplitude ratio.

7. Conclusion

The most significant conclusion which emerges out of numerical discussion is that impedence parameter and relaxation time plays a vital role in propgation of waves . It is also observed that impedence parameter plays predominant role in case of CT-theory of thermoelasticity than that observed for L-S theory. The model presented in the problem is useful in investigations concerned with earthquake and other phenomenon in seismology. It is also concluded that reflection is influenced by the presence of diffusion parameter, which is useful for more realstic model of elastic diffusion present in the earth interior.

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