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Nonrigid Group Theory of Water Clusters (Cyclic Forms): $(H_2O)_i$ for $2 \leq i \leq 6$

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Abstract. The character table of the fully nonrigid water cluster (cyclic forms), $(H_2O)_i$, with C_{ih} symmetry derived for the first time, for $2 \leq i \leq 6$. The group of all feasible permutations is the wreath product of groups $S_i[S_2]$ which consists of $i!2^i$ operations for $i = 2, ..., 6$ divided into (w.r.t) 5, 10, 20, 36, 65 conjugacy classes and 5, 10, 20, 36, 65 irreducible representations respectively. We compute the full character table of $(H_2O)_2, (H_2O)_3, (H_2O)_4, (H_2O)_5$ and $(H_2O)_6$.

Keywords: Nonrigid Group Theory, Symmetry, Wreath Product, Conjugacy Classes, Character Table, Water Cluster.

2000 Mathematics subject classification: 92E10, 20C15, 20C40, 20C30.

1. Introduction

Although the extent of tunneling would depend on the actual barriers, there is a compelling need to consider the molecular symmetry groups of the nonrigid cluster from semirigid to fully nonrigid limits. Longuet-Higgins [1] has formulated the symmetry groups of nonrigid molecules as permutation-inversion groups by including all feasible permutation of the nuclei under such fluxional or tunneling motions. Up to now, the character table of the fully nonrigid $(H_2O)_i$ with C_{ih} symmetry for $i = 2, ..., 6$ has not been obtained. Balasubramanian [2-8] has shown that the groups of nonrigid molecules can be expressed as wreath product

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and generalized wreath product groups. These groups have also been used in a number of chemical applications such as enumeration of isomers [9-12], weakly bound van der Waals, or hydrogen-bonded complexes such as $(NH_3)_2, (NH_3)_4, (C_6H_6)_2$, etc. [1, 13-17], polyhedral structures [18,19], spectroscopy [14-17,20], and cluster [21]. King [18,19] has applied the wreath product groups to represent the symmetries of four-dimensional analogues of polyhedra, Thus, apart from the current motivation of calculating the fully nonrigid $(H_2O)_i$ for $2 \leq i \leq 6$, there is considerable interest in wreath product groups of higher order and their character tables. Balasubramanian [5] has applied combinatorial methods without the construction of the character tables for the spin statistics of protonated forms of water cluster. In this study, we have derived the character table of the nonrigid $(H_2O)_i$ for $2 \leq i \leq 6$ in its full nonrigid limit. The resulting group is shown to be the wreath product $S_i[S_2]$, for $2 \leq i \leq 6$ where the group S_n is a permutation group of $n!$ operation, and the square bracket symbol stands for wreath products.

We show that the fully nonrigid $(H_2O)_i$ with C_{ih} symmetry exhibits a group of $i!2^i$ op-

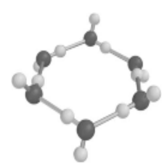

FIGURE 1. $(H_2O)_6$

erations for $i = 2, ..., 6$ divided into 5, 10, 20, 36, 65 conjugacy classes and 5, 10, 20, 36, 65 irreducible representations respectively. We have obtained the character tables of these groups.

2. Wreath Product of the Group $S_i[S_2]$ for Water Clusters $(H_2O)_i$, $2 \leq i \leq 6$

Although the theory of wreath product groups and related mathematical details have been described in sufficient details elsewhere [3, 6]. We provide the salient points so that this work on $(H_2O)_i$, $2 \leq i \leq 6$ is sufficiently self contained. Suppose that G is the group of permutations of the oxygen nuclei in the fully nonrigid limit where they are allowed to exchange and H is the group of permutations of the protons owing to the facile flipping motion. Thus G is the set of $i!$ permutations of i oxygen nuclei, and H is the group S_2 of protons on each Water molecule that corresponds to the flipping motion which exchanges these protons. In general, the permutation group S_n [22-24] consists of $n!$ permutations of n objects of a set of chosen nuclei, denoted by Ω to represent the rigid framework. Note that the notation S_n that we use here differs from the point group S_n that corresponds to n fold improper axis of rotation. All references to S_n in this work mean the permutation group of $n!$ operations. As the oxygen atoms get permuted, they carry the protons attached to them, and so induce a permutation of the protons. Consequently, the overall group of $(H_2O)_i$ becomes the wreath product of G with H, denoted by $G[H]$, which becomes $S_i[S_2]$ in this case. The wreath product group $G[H]$ is defined as the set of permutations

$$\{\ (g;\pi)|\pi \quad \text{is a mapping of } \Omega \text{ into } H, g \in G\}$$

and the product of two permutations is defined by

$$(g;\pi)(g^{'};\pi^{'}) = (gg^{'};\pi\pi_g^{'})$$

where

$$\pi_g(i) = \pi(g^{-1}i), \forall i \in \Omega$$
$$\pi\pi^{'}(i) = \pi(i)\pi^{'}(i), \forall i \in \Omega$$

An element of $G[H]$ is represented by $(g; h_1, h_2, ..., h_n)$, where $g \in G$ and $h_i \in H$. Thus, the group $G[H]$ contains $|G||H|^n$ elements where n is order of Ω. In the case of $(H_2O)_i$, the order of the full nonrigid permutation group is given by

$$|S_i[S_2]| = i!(2)^i$$

The group $S_i[S_2]$ is isomorphic with

$$S_i[S_2] = (S_2 \times ... \times S_2) \wedge S_i^{'}$$

where the symbols $\times$ and $\wedge$ stand for direct and semidirect product, respectively.

3. Conjugacy Classes

Let $S_n[H]$ be the group under consideration and $(g;\pi)$ be an element of $S_n[H]$. If we adopt the convention to begin each cyclic factor with the least symbol included in the cycle decomposition of g, then we can associate with each cyclic factor $[j; g(j), g^2(j), ..., g^r(j)]$ of g the unique element $\pi\pi_g\pi_{g^2}...\pi_{g^r}(j) = \pi(j)\pi[g^{-1}(j)]...\pi[g^{-r}(j)]$ in S_n. Let us call this element the cyclic product associated with $[j; g(j), g^2(j), ..., g^r(j)]$ with respect to π. Let the permutation $g \in S_n$ be of the type $T_g = (a_1, a_2, ..., a_n)$ (where a_i denotes cycles of length i). There are a_k cycle products (defined above) associated with the the a_k cycles of length k of g with respect to π. Let $C_1, C_2, ..., C_s$ be the conjugacy classes of H. If exactly a_{ik} of these cycle products belong to C_i, then the $s \times n$ matrix defined below is the cycle type of an element $(g;\pi)$ of the wreath product $T(g;\pi) = a_{ik}(1 \le i \le s, 1 \le k \le n)$.
Let $P(m)$ denote the number of partitions of the integer m, with the convention that $P(0) = 1$. Let n be partitioned into the ordered s-tuples $(n) = (n_1, n_2, ..., n_s)$ such that $\sum_i n_i = n$. (Recall that s is the number of conjugacy classes of H). Then the number of conjugacy classes of $S_n[H]$ is

$$\sum_{(n)} P(n_1)P(n_2)...P(n_s).$$

For a proof see Kerber [25]. The order of the conjugacy class whose matrix type is (a_{ik}) [26] is given by

$$\frac{|S_n[H]|}{\prod_{i,k} a_{ik}!(k.|H|/|C_i|)^{a_{ik}}}.$$

Therefore, we can compute the conjugacy classes of $S_i[S_2]$ for $i = 2, 3, 4, 5, 6$ which are shown in the following Tables .

Table 1: conjugacy classes of $S_2[S_2]$

No	Class Representation	Order	Symbole
1	Identity	1	1a
2	(3,4)	2	2a
3	(1,2)(3,4)	1	2b
4	(1,3)(2,4)	2	2c
5	(1,3,2,4)	2	4a

Table 2: conjugacy classes of $S_3[S_2]$

No	Class Representation	Order	Symbole
1	Identity	1	1a
2	(5,6)	3	2a
3	(3,4)(5,6)	3	2b
4	(3,5)(4,6)	6	2c
5	(3,5,4,6)	6	2d
6	(1,2)(3,4)(5,6)	1	2e
7	(1,2)(3,5)(4,6)	6	2f
8	(1,2)(3,5,4,6)	6	4a
9	(1,3,5)(2,4,6)	8	3a
10	(1,3,5,2,4,6)	8	6a

Table 3: conjugacy classes of $S_4[S_2]$

No	Class Representation	Order	Symbole
1	Identity	1	1a
2	(7,8)	4	2a
3	(5,6)(7,8)	6	2b
4	(5,7)(6,8)	12	2c
5	(5,7,6,8)	12	4a
6	(3,4)(5,6)(7,8)	4	2d
7	(3,4)(5,7)(6,8)	24	2e
8	(3,4)(5,7,6,8)	24	4b
9	(3,5,7)(4,6,8)	32	3a
10	(3,5,7,4,6,8)	32	3b
11	(1,2)(3,4)(5,6)(7,8)	1	2f
12	(1,2)(3,4)(5,7)(6,8)	12	2g
13	(1,2)(3,4)(5,7,6,8)	12	2h
14	(1,2)(3,5,7)(4,6,8)	32	6a
15	(1,2)(3,5,7,4,6,8)	32	6b
16	(1,3)(2,4)(5,7)(6,8)	12	2i
17	(1,3)(2,4)(5,7,6,8)	24	4c
18	(1,3,2,4)(5,7,6,8)	12	4d
19	(1,3,5,7)(2,4,6,8)	48	4e
20	(1,3,5,7,2,4,6,8)	48	8a

Table 4: conjugacy classes of $S_5[S_2]$

No	Class Representation	Order	Symbole
1	Identity	1	1a
2	(9,10)	5	2a
3	(7,8)(9,10)	10	2b
4	(5,6)(7,8)(9,10)	10	2c
5	(3,4)(5,6)(7,8)(9,10)	5	2d
6	(1,2)(3,4)(5,6)(7,8)(9,10)	1	2e
7	(7,9)(8,10)	20	2f
8	(7,10,8,9)	20	4a
9	(5,6)(7,9)(8,10)	60	2g
10	(5,6)(7,10,8,9)	60	4b
11	(3,4)(5,6)(7,9)(8,10)	60	2h
12	(3,4)(5,6)(7,10,8,9)	60	4c
13	(1,2)(3,4)(5,6)(7,9)(8,10)	20	2i
14	(1,2)(3,4)(5,6)(7,10,8,9)	20	4d
15	(5,7,9)(6,8,10)	80	3a
16	(5,7,10,6,8,9)	80	6a
17	(3,4)(5,7,9)(6,8,10)	160	6b
18	(3,4)(5,7,10,6,8,9)	160	6c
19	(1,2)(3,4)(5,7,9)(6,8,10)	80	6d
20	(1,2)(3,4)(5,7,10,6,8,9)	80	6e
21	(3,5)(4,6)(7,9)(8,10)	60	2j
22	(3,5)(4,6)(7,10,8,9)	120	4e
23	(3,6,4,5)(7,10,8,9)	60	4f
24	(1,2)(3,5)(4,6)(7,9)(8,10)	60	2k
25	(1,2)(3,5)(4,6)(7,10,8,9)	120	4g
26	(1,2)(3,6,4,5)(7,10,8,9)	60	4h
27	(3,5,7,9)(4,6,8,10)	240	4i
28	(3,5,7,10,4,6,8,9)	240	8a
29	(1,2)(3,5,7,9)(4,6,8,10)	240	4j
30	(1,2)(3,5,7,10,4,6,8,9)	240	8b
31	(1,3)(2,4)(5,7,9)(6,8,10)	160	6f
32	(1,3)(2,4)(5,7,10,6,8,9)	160	6g
33	(1,4,2,3)(5,7,9)(6,8,10)	160	12a
34	(1,4,2,3)(5,7,10,6,8,9)	160	12b
35	(1,3,5,7,9)(2,4,6,8,10)	384	5a
36	(1,3,5,7,10,2,4,6,8,9)	384	10a

Table 5: conjugacy classes of $S_6[S_2]$

No	Class Representation	Order	Symbole
1	()	1	1a
2	(11,12)	6	2a
3	(9,10)(11,12)	15	2b
4	(7,8)(9,10)(11,12)	20	2c
5	(5,6)(7,8)(9,10)(11,12)	15	2d
6	(3,4)(5,6)(7,8)(9,10)(11,12)	6	2e
7	(1,2)(3,4)(5,6)(7,8)(9,10)(11,12)	1	2f
8	(9,11)(10,12)	30	2g
9	(9,12,10,11)	30	4a
10	(7,8)(9,11)(10,12)	120	2h
11	(7,8)(9,12,10,11)	120	4b
12	(5,6)(7,8)(9,11)(10,12)	180	2i
13	(5,6)(7,8)(9,12,10,11)	180	4c
14	(3,4)(5,6)(7,8)(9,11)(10,12)	120	2j
15	(3,4)(5,6)(7,8)(9,12,10,11)	120	4d
16	(1,2)(3,4)(5,6)(7,8)(9,11)(10,12)	30	2k
17	(1,2)(3,4)(5,6)(7,8)(9,12,10,11)	30	4e
18	(7,9,11)(8,10,12)	160	3a
19	(7,9,12,8,10,11)	160	6a
20	(5,6)(7,9,11)(8,10,12)	480	6b
21	(5,6)(7,9,12,8,10,11)	480	6c
22	(3,4)(5,6)(7,9,11)(8,10,12)	480	6d
23	(3,4)(5,6)(7,9,12,8,10,11)	480	6e
24	(1,2)(3,4)(5,6)(7,9,11)(8,10,12)	160	6f
25	(1,2)(3,4)(5,6)(7,9,12,8,10,11)	160	6g
26	(5,7)(6,8)(9,11)(10,12)	180	2l
27	(5,7)(6,8)(9,12,10,11)	360	4f
28	(5,8,6,7)(9,12,10,11)	180	4g
29	(3,4)(5,7)(6,8)(9,11)(10,12)	360	2m
30	(3,4)(5,7)(6,8)(9,12,10,11)	720	4h
31	(3,4)(5,8,6,7)(9,12,10,11)	360	4i
32	(1,2)(3,4)(5,7)(6,8)(9,11)(10,12)	180	2n
33	(1,2)(3,4)(5,7)(6,8)(9,12,10,11)	360	4j
34	(1,2)(3,4)(5,8,6,7)(9,12,10,11)	180	4k
35	(5,7,9,11)(6,8,10,12)	720	4l

Continue of Table 5

No	Class Representation	Order	Symbole
36	(5,7,9,12,6,8,10,11)	720	8a
37	(3,4)(5,7,9,11)(6,8,10,12)	1440	4m
38	(3,4)(5,7,9,12,6,8,10,11)	1440	8b
39	(1,2)(3,4)(5,7,9,11)(6,8,10,12)	720	4n
40	(1,2)(3,4)(5,7,9,12,6,8,10,11)	720	8c
41	(3,5)(4,6)(7,9,11)(8,10,12)	960	6h
42	(3,5)(4,6)(7,9,12,8,10,11)	960	6i
43	(3,6,4,5)(7,9,11)(8,10,12)	960	12a
44	(3,6,4,5)(7,9,12,8,10,11)	960	12b
45	(1,2)(3,5)(4,6)(7,9,11)(8,10,12)	960	6j
46	(1,2)(3,5)(4,6)(7,9,12,8,10,11)	960	6k
47	(1,2)(3,6,4,5)(7,9,11)(8,10,12)	960	12c
48	(1,2)(3,6,4,5)(7,9,12,8,10,11)	960	12d
49	(3,5,7,9,11)(4,6,8,10,12)	2304	5a
50	(3,5,7,9,12,4,6,8,10,11)	2304	10a
51	(1,2)(3,5,7,9,11)(4,6,8,10,12)	2304	10b
52	(1,2)(3,5,7,9,12,4,6,8,10,11)	2304	10c
53	(1,3)(2,4)(5,7)(6,8)(9,11)(10,12)	120	2o
54	(1,3)(2,4)(5,7)(6,8)(9,12,10,11)	360	4o
55	(1,3)(2,4)(5,8,6,7)(9,12,10,11)	360	4p
56	(1,4,2,3)(5,8,6,7)(9,12,10,11)	120	4q
57	(1,3)(2,4)(5,7,9,11)(6,8,10,12)	1440	4r
58	(1,3)(2,4)(5,7,9,12,6,8,10,11)	1440	8d
59	(1,4,2,3)(5,7,9,11)(6,8,10,12)	1440	4s
60	(1,4,2,3)(5,7,9,12,6,8,10,11)	1440	8e
61	(1,3,5)(2,4,6)(7,9,11)(8,10,12)	640	3b
62	(1,3,5)(2,4,6)(7,9,12,8,10,11)	1280	6l
63	(1,3,6,2,4,5)(7,9,12,8,10,11)	640	6m
64	(1,3,5,7,9,11)(2,4,6,8,10,12)	3840	6n
65	(1,3,5,7,9,12,2,4,6,8,10,11)	3840	12e

4. Character Tables of Wreath Product Groups of $S_i[S_2]$, $2 \leq i \leq 6$

We compute character tables of above groups by GAP[27], and developed following programm for GAP, and we run this programm for $i = 2, 3, 4, 5, 6$, and we obtained character table of any nonrigid group

```
gap> si := SymmetricGroup(i);
gap>s2 := SymmetricGroup(2);
gap> g := WreathProduct(s2,si);
gap> Display(CharacterTable(g));
```

Table 6: character table of $S_2[S_2]$

	1a	2a	2b	2c	4a
χ_1	1	1	1	1	1
χ_2	1	-1	1	1	-1
χ_3	1	1	1	-1	-1
χ_4	1	-1	1	-1	1
χ_5	2	0	-2	0	0

Table 7: character table of $S_3[S_2]$

	1a	2a	2b	2c	4a	2d	2e	4b	3a	6a
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	-1	-1	1	1	-1
χ_3	1	1	1	-1	-1	1	-1	-1	1	1
χ_4	1	-1	1	-1	1	-1	1	-1	1	-1
χ_5	2	2	2	0	0	2	0	0	-1	-1
χ_6	2	-2	2	0	0	-2	0	0	-1	1
χ_7	3	-1	-1	-1	1	3	-1	1	0	0
χ_8	3	-1	-1	1	-1	3	1	-1	0	0
χ_9	3	1	-1	-1	-1	-3	1	1	0	0
χ_{10}	3	1	-1	1	1	-3	-1	-1	0	0

Table 8: character table of $S_4[S_2]$

	1a	2a	2b	2c	4a	2d	2e	4b	3a	6a	2f	2g	4c	6b	6c	2h	4d	4e	4f	8a
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	-1	-1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	-1
χ_3	1	1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	1	1	1	1	1	-1	-1
χ_4	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1
χ_5	2	-2	2	0	0	-2	0	0	-1	1	2	0	0	1	-1	2	-2	2	0	0
χ_6	2	2	2	0	0	2	0	0	-1	-1	2	0	0	-1	-1	2	2	2	0	0
χ_7	3	-3	3	-1	1	-3	1	-1	0	0	3	-1	1	0	0	-1	1	-1	1	-1
χ_8	3	-3	3	1	-1	-3	-1	1	0	0	3	1	-1	0	0	-1	1	-1	-1	1
χ_9	3	3	3	-1	-1	3	-1	-1	0	0	3	-1	-1	0	0	-1	-1	-1	1	1
χ_{10}	3	3	3	1	1	3	1	1	0	0	3	1	1	0	0	-1	-1	-1	-1	-1
χ_{11}	4	-2	0	-2	2	2	0	0	1	-1	-4	2	-2	1	-1	0	0	0	0	0
χ_{12}	4	-2	0	2	-2	2	0	0	1	-1	-4	-2	2	1	-1	0	0	0	0	0
χ_{13}	4	2	0	-2	-2	-2	0	0	1	1	-4	2	2	-1	-1	0	0	0	0	0
χ_{14}	4	2	0	2	2	-2	0	0	1	1	-4	-2	-2	-1	-1	0	0	0	0	0
χ_{15}	6	0	-2	2	0	0	0	-2	0	0	6	2	0	0	0	2	0	-2	0	0
χ_{16}	6	0	-2	-2	0	0	0	2	0	0	6	-2	0	0	0	2	0	-2	0	0
χ_{17}	6	0	-2	0	-2	0	2	0	0	0	6	0	-2	0	0	-2	0	2	0	0
χ_{18}	6	0	-2	0	2	0	-2	0	0	0	6	0	2	0	0	-2	0	2	0	0
χ_{19}	8	-4	0	0	0	4	0	0	-1	1	-8	0	0	-1	1	0	0	0	0	0
χ_{20}	8	4	0	0	0	-4	0	0	-1	-1	-8	0	0	1	1	0	0	0	0	0

Table 9: character table of $S_5[S_2]$

	1a	2a	2b	2c	2d	2e	2f	4a	2g	4b	2h	4c	2i	4d	3a	6a	6b	6c	6d	6e	2j	4e	4f
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	1
χ_3	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
χ_4	1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	1
χ_5	4	-4	4	-4	4	-4	2	-2	-2	2	2	-2	-2	2	1	-1	-1	1	1	-1	0	0	0
χ_6	4	4	4	4	4	4	2	2	2	2	2	2	2	2	1	1	1	1	1	1	0	0	0
χ_7	4	-4	4	-4	4	-4	-2	2	2	-2	-2	2	2	-2	1	-1	-1	1	1	-1	0	0	0
χ_8	4	4	4	4	4	4	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	0	0	0
χ_9	5	-5	5	-5	5	-5	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1
χ_{10}	5	5	5	5	5	5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1
χ_{11}	5	-5	5	-5	5	-5	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1
χ_{12}	5	5	5	5	5	5	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1
χ_{13}	5	-3	1	1	-3	5	3	-3	-1	1	-1	1	3	-3	2	-2	0	0	-2	2	1	-1	1
χ_{14}	5	3	1	-1	-3	-5	3	3	1	1	-1	-1	-3	-3	2	2	0	0	-2	-2	1	1	1
χ_{15}	5	-3	1	1	-3	5	-3	3	1	-1	1	-1	-3	3	2	-2	0	0	-2	2	1	-1	1
χ_{16}	5	3	1	-1	-3	-5	-3	-3	-1	-1	1	1	3	3	2	2	0	0	-2	-2	1	1	1
χ_{17}	6	-6	6	-6	6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	-2
χ_{18}	6	6	6	6	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2
χ_{19}	10	2	-2	-2	2	10	-2	-4	2	0	2	0	-2	-4	1	1	-1	-1	1	1	-2	0	2
χ_{20}	10	-2	-2	2	2	-10	-2	4	-2	0	2	0	2	-4	1	-1	1	-1	1	-1	-2	0	2
χ_{21}	10	2	-2	-2	2	10	2	4	-2	0	-2	0	2	4	1	1	-1	-1	1	1	-2	0	2
χ_{22}	10	-2	-2	2	2	-10	2	-4	2	0	-2	0	-2	4	1	-1	1	-1	1	-1	-2	0	2
χ_{23}	10	2	-2	-2	2	10	4	2	0	-2	0	-2	4	2	1	1	-1	-1	1	1	2	0	-2
χ_{24}	10	-2	-2	2	2	-10	4	-2	0	-2	0	2	-4	2	1	-1	1	-1	1	-1	2	0	-2
χ_{25}	10	2	-2	-2	2	10	-4	-2	0	2	0	2	-4	-2	1	1	-1	-1	1	1	2	0	-2
χ_{26}	10	-2	-2	2	2	-10	-4	2	0	2	0	-2	4	-2	1	-1	1	-1	1	-1	2	0	-2
χ_{27}	10	-6	2	2	-6	10	0	0	0	0	0	0	0	0	-2	2	0	0	2	-2	2	-2	2
χ_{28}	10	6	2	-2	-6	-10	0	0	0	0	0	0	0	0	-2	-2	0	0	2	2	2	2	2
χ_{29}	15	-9	3	3	-9	15	-3	3	1	-1	1	-1	-3	3	0	0	0	0	0	0	-1	1	-1
χ_{30}	15	9	3	-3	-9	-15	-3	-3	-1	-1	1	1	3	3	0	0	0	0	0	0	-1	-1	-1
χ_{31}	15	-9	3	3	-9	15	3	-3	-1	1	-1	1	3	-3	0	0	0	0	0	0	-1	1	-1
χ_{32}	15	9	3	-3	-9	-15	3	3	1	1	-1	-1	-3	-3	0	0	0	0	0	0	-1	-1	-1
χ_{33}	20	4	-4	-4	4	20	-2	2	-2	2	-2	2	-2	2	-1	-1	1	1	-1	-1	0	0	0
χ_{34}	20	-4	-4	4	4	-20	-2	-2	2	2	-2	-2	2	2	-1	1	-1	1	-1	1	0	0	0
χ_{35}	20	4	-4	-4	4	20	2	-2	2	-2	2	-2	2	-2	-1	-1	1	1	-1	-1	0	0	0
χ_{36}	20	-4	-4	4	4	-20	2	2	-2	-2	2	2	-2	-2	-1	1	-1	1	-1	1	0	0	0

Continue of Table 9

	2k	4g	4h	4i	8a	4j	8b	6f	6g	12a	12b	5a	10a
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1
χ_3	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1
χ_4	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1
χ_5	0	0	0	0	0	0	0	-1	1	1	-1	-1	1
χ_6	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
χ_7	0	0	0	0	0	0	0	1	-1	-1	1	-1	1
χ_8	0	0	0	0	0	0	0	1	1	1	1	-1	-1
χ_9	-1	1	-1	1	-1	-1	1	-1	1	1	-1	0	0
χ_{10}	1	1	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_{11}	-1	1	-1	-1	1	1	-1	1	-1	-1	1	0	0
χ_{12}	1	1	1	-1	-1	-1	-1	1	1	1	1	0	0
χ_{13}	1	-1	1	1	-1	1	-1	0	0	0	0	0	0
χ_{14}	-1	-1	-1	1	1	-1	-1	0	0	0	0	0	0
χ_{15}	1	-1	1	-1	1	-1	1	0	0	0	0	0	0
χ_{16}	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_{17}	2	-2	2	0	0	0	0	0	0	0	0	1	-1
χ_{18}	-2	-2	-2	0	0	0	0	0	0	0	0	1	1
χ_{19}	-2	0	2	0	0	0	0	1	1	-1	-1	0	0
χ_{20}	2	0	-2	0	0	0	0	1	-1	1	-1	0	0
χ_{21}	-2	0	2	0	0	0	0	-1	-1	1	1	0	0
χ_{22}	2	0	-2	0	0	0	0	-1	1	-1	1	0	0
χ_{23}	2	0	-2	0	0	0	0	1	1	-1	-1	0	0
χ_{24}	-2	0	2	0	0	0	0	1	-1	1	-1	0	0
χ_{25}	2	0	-2	0	0	0	0	-1	-1	1	1	0	0
χ_{26}	-2	0	2	0	0	0	0	-1	1	-1	1	0	0
χ_{27}	2	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{28}	-2	-2	-2	0	0	0	0	0	0	0	0	0	0
χ_{29}	-1	1	-1	1	-1	1	-1	0	0	0	0	0	0
χ_{30}	1	1	1	1	1	-1	-1	0	0	0	0	0	0
χ_{31}	-1	1	-1	-1	1	-1	1	0	0	0	0	0	0
χ_{32}	1	1	1	-1	-1	1	1	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{34}	0	0	0	0	0	0	0	1	-1	1	-1	0	0
χ_{35}	0	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{36}	0	0	0	0	0	0	0	-1	1	-1	1	0	0

Table 10: character table of $S_6[S_2]$

	1a	2a	2b	2c	2d	2e	2f	2g	4a	2h	4b	2i	4c	2j	4d	2k	4e	3a	6a	6b	6c
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
χ_3	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_4	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
χ_5	5	5	5	5	5	5	5	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	2	2	2
χ_6	5	-5	5	-5	5	-5	5	-3	3	3	-3	-3	3	3	-3	-3	3	2	-2	-2	2
χ_7	5	5	5	5	5	5	5	3	3	3	3	3	3	3	3	3	3	2	2	2	2
χ_8	5	-5	5	-5	5	-5	5	3	-3	-3	3	3	-3	-3	3	3	-3	2	-2	-2	2
χ_9	5	-5	5	-5	5	-5	5	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1
χ_{10}	5	-5	5	-5	5	-5	5	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
χ_{11}	5	5	5	5	5	5	5	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{12}	5	5	5	5	5	5	5	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
χ_{13}	6	4	2	0	-2	-4	-6	4	4	2	2	0	0	-2	-2	-4	-4	3	3	1	1
χ_{14}	6	-4	2	0	-2	4	-6	4	-4	-2	2	0	0	2	-2	-4	4	3	-3	-1	1
χ_{15}	6	4	2	0	-2	-4	-6	-4	-4	-2	-2	0	0	2	2	4	4	3	3	1	1
χ_{16}	6	-4	2	0	-2	4	-6	-4	4	2	-2	0	0	-2	2	4	-4	3	-3	-1	1
χ_{17}	9	9	9	9	9	9	9	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	0
χ_{18}	9	-9	9	-9	9	-9	9	-3	3	3	-3	-3	3	3	-3	-3	3	0	0	0	0
χ_{19}	9	9	9	9	9	9	9	3	3	3	3	3	3	3	3	3	3	0	0	0	0
χ_{20}	9	-9	9	-9	9	-9	9	3	-3	-3	3	3	-3	-3	3	3	-3	0	0	0	0
χ_{21}	10	-10	10	-10	10	-10	10	-2	2	2	-2	-2	2	2	-2	-2	2	1	-1	-1	1
χ_{22}	10	-10	10	-10	10	-10	10	2	-2	-2	2	2	-2	-2	2	2	-2	1	-1	-1	1
χ_{23}	10	10	10	10	10	10	10	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1
χ_{24}	10	10	10	10	10	10	10	2	2	2	2	2	2	2	2	2	2	1	1	1	1
χ_{25}	15	-5	-1	3	-1	-5	15	-5	7	-1	-1	3	-1	-1	-1	-5	7	3	-3	1	-1
χ_{26}	15	5	-1	-3	-1	5	15	-5	-7	1	-1	3	1	1	-1	-5	-7	3	3	-1	-1
χ_{27}	15	-5	-1	3	-1	-5	15	5	-7	1	1	-3	1	1	1	5	-7	3	-3	1	-1
χ_{28}	15	5	-1	-3	-1	5	15	5	7	-1	1	-3	-1	-1	1	5	7	3	3	-1	-1
χ_{29}	15	-5	-1	3	-1	-5	15	-7	5	1	1	1	-3	1	1	-7	5	3	-3	1	-1
χ_{30}	15	5	-1	-3	-1	5	15	-7	-5	-1	1	1	3	-1	1	-7	-5	3	3	-1	-1
χ_{31}	15	-5	-1	3	-1	-5	15	7	-5	-1	-1	-1	3	-1	-1	7	-5	3	-3	1	-1
χ_{32}	15	5	-1	-3	-1	5	15	7	5	1	-1	-1	-3	1	-1	7	5	3	3	-1	-1
χ_{33}	16	16	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2
χ_{34}	16	-16	16	-16	16	-16	16	0	0	0	0	0	0	0	0	0	0	-2	2	2	-2
χ_{35}	20	0	-4	0	4	0	-20	-8	0	0	4	0	0	0	-4	8	0	2	0	0	-2

Continue of Table 10

	1a	2a	2b	2c	2d	2e	2f	2g	4a	2h	4b	2i	4c	2j	4d	2k	4e	3a	6a	6b	6c
χ_{36}	20	0	-4	0	4	0	-20	8	0	0	-4	0	0	0	4	-8	0	2	0	0	-2
χ_{37}	20	0	-4	0	4	0	-20	0	-8	4	0	0	0	-4	0	0	8	2	0	0	-2
χ_{38}	20	0	-4	0	4	0	-20	0	8	-4	0	0	0	4	0	0	-8	2	0	0	-2
χ_{39}	24	16	8	0	-8	-16	-24	-8	-8	-4	-4	0	0	4	4	8	8	3	3	1	1
χ_{40}	24	-16	8	0	-8	16	-24	-8	8	4	-4	0	0	-4	4	8	-8	3	-3	-1	1
χ_{40}	24	16	8	0	-8	-16	-24	8	8	4	4	0	0	-4	-4	-8	-8	3	3	1	1
χ_{42}	24	-16	8	0	-8	16	-24	8	-8	-4	4	0	0	4	-4	-8	8	3	-3	-1	1
χ_{43}	30	20	10	0	-10	-20	-30	4	4	2	2	0	0	-2	-2	-4	-4	-3	-3	-1	-1
χ_{44}	30	-20	10	0	-10	20	-30	4	-4	-2	2	0	0	2	-2	-4	4	-3	3	1	-1
χ_{45}	30	20	10	0	-10	-20	-30	-4	-4	-2	-2	0	0	2	2	4	4	-3	-3	-1	-1
χ_{46}	30	-20	10	0	-10	20	-30	-4	4	2	-2	0	0	-2	2	4	-4	-3	3	1	-1
χ_{47}	30	-10	-2	6	-2	-10	30	-2	-2	2	2	-2	-2	2	2	-2	-2	-3	3	-1	1
χ_{48}	30	-10	-2	6	-2	-10	30	2	2	-2	-2	2	2	-2	-2	2	2	-3	3	-1	1
χ_{49}	30	10	-2	-6	-2	10	30	-2	2	-2	2	-2	2	-2	2	-2	2	-3	-3	1	1
χ_{50}	30	10	-2	-6	-2	10	30	2	-2	2	-2	2	-2	2	-2	2	-2	-3	-3	1	1
χ_{51}	36	24	12	0	-12	-24	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	36	-24	12	0	-12	24	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	40	0	-8	0	8	0	-40	-8	-8	4	4	0	0	-4	-4	8	8	1	3	-3	-1
χ_{54}	40	0	-8	0	8	0	-40	-8	8	-4	4	0	0	4	-4	8	-8	1	-3	3	-1
χ_{55}	40	0	-8	0	8	0	-40	8	-8	4	-4	0	0	-4	4	-8	8	1	-3	3	-1
χ_{56}	40	0	-8	0	8	0	-40	8	8	-4	-4	0	0	4	4	-8	-8	1	3	-3	-1
χ_{57}	45	-15	-3	9	-3	-15	45	9	-3	-3	-3	1	5	-3	-3	9	-3	0	0	0	0
χ_{58}	45	15	-3	-9	-3	15	45	9	3	3	-3	1	-5	3	-3	9	3	0	0	0	0
χ_{59}	45	-15	-3	9	-3	-15	45	-9	3	3	3	-1	-5	3	3	-9	3	0	0	0	0
χ_{60}	45	15	-3	-9	-3	15	45	-9	-3	-3	3	-1	5	-3	3	-9	-3	0	0	0	0
χ_{61}	45	-15	-3	9	-3	-15	45	-3	9	-3	-3	5	1	-3	-3	-3	9	0	0	0	0
χ_{62}	45	-15	-3	9	-3	-15	45	3	-9	3	3	-5	-1	3	3	3	-9	0	0	0	0
χ_{63}	45	15	-3	-9	-3	15	45	-3	-9	3	-3	5	-1	3	-3	-3	-9	0	0	0	0
χ_{64}	45	15	-3	-9	-3	15	45	3	9	-3	3	-5	1	-3	3	3	9	0	0	0	0
χ_{65}	80	0	-16	0	16	0	-80	0	0	0	0	0	0	0	0	0	0	-4	0	0	4

Continue of Table 10

	6d	6e	6f	6g	2l	4f	4g	2m	4h	4i	2n	4j	4k	4l	8a	4m	8b	4n	8c	6h	6i	12a	12b
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_4	1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1
χ_5	2	2	2	2	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0
χ_6	2	-2	-2	2	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	0	0	0	0
χ_7	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
χ_8	2	-2	-2	2	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	0	0	0	0
χ_9	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
χ_{10}	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1
χ_{11}	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
χ_{12}	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{13}	-1	-1	-3	-3	2	2	2	0	0	0	-2	-2	-2	2	2	0	0	-2	-2	1	1	1	1
χ_{14}	-1	1	3	-3	2	-2	2	0	0	0	-2	2	-2	2	-2	0	0	-2	2	1	-1	-1	1
χ_{15}	-1	-1	-3	-3	2	2	2	0	0	0	-2	-2	-2	-2	-2	0	0	2	2	-1	-1	-1	-1
χ_{16}	-1	1	3	-3	2	-2	2	0	0	0	-2	2	-2	-2	2	0	0	2	-2	-1	1	1	-1
χ_{17}	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
χ_{18}	0	0	0	0	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	0	0	0	0
χ_{19}	0	0	0	0	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0
χ_{20}	0	0	0	0	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	0	0	0	0
χ_{21}	1	-1	-1	1	-2	2	-2	2	-2	2	-2	2	-2	0	0	0	0	0	0	1	-1	-1	1
χ_{22}	1	-1	-1	1	-2	2	-2	2	-2	2	-2	2	-2	0	0	0	0	0	0	-1	1	1	-1
χ_{23}	1	1	1	1	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	1	1	1	1
χ_{24}	1	1	1	1	-2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0	0	0	-1	-1	-1	-1
χ_{25}	-1	1	-3	3	-1	-1	3	3	-1	-1	-1	-1	3	-1	1	-1	1	-1	1	1	-1	1	-1
χ_{26}	-1	-1	3	3	-1	1	3	-3	-1	1	-1	1	3	-1	-1	1	1	-1	-1	1	1	-1	-1
χ_{27}	-1	1	-3	3	-1	-1	3	3	-1	-1	-1	-1	3	1	-1	1	-1	1	-1	-1	1	-1	1
χ_{28}	-1	-1	3	3	-1	1	3	-3	-1	1	-1	1	3	1	1	-1	-1	1	1	-1	-1	1	1
χ_{29}	-1	1	-3	3	3	-1	-1	-1	-1	3	3	-1	-1	-1	1	-1	1	-1	1	-1	1	-1	1
χ_{30}	-1	-1	3	3	3	1	-1	1	-1	-3	3	1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1
χ_{31}	-1	1	-3	3	3	-1	-1	-1	-1	3	3	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{32}	-1	-1	3	3	3	1	-1	1	-1	-3	3	1	-1	1	1	-1	-1	1	1	1	1	-1	-1
χ_{33}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	2	0	0	-2	4	0	-4	0	0	0	-4	0	4	0	0	0	0	0	0	-2	0	0	2

Continue of Table 10

	6d	6e	6f	6g	2l	4f	4g	2m	4h	4i	2n	4j	4k	4l	8a	4m	8b	4n	8c	6h	6i	12a	12b
χ_{36}	2	0	0	-2	4	0	-4	0	0	0	-4	0	4	0	0	0	0	0	0	2	0	0	-2
χ_{37}	2	0	0	-2	-4	0	4	0	0	0	4	0	-4	0	0	0	0	0	0	0	2	-2	0
χ_{38}	2	0	0	-2	-4	0	4	0	0	0	4	0	-4	0	0	0	0	0	0	0	-2	2	0
χ_{39}	-1	-1	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_{40}	-1	1	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1
χ_{41}	-1	-1	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{42}	-1	1	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1
χ_{43}	1	1	3	3	2	2	2	0	0	0	-2	-2	-2	-2	-2	0	0	2	2	1	1	1	1
χ_{44}	1	-1	-3	3	2	-2	2	0	0	0	-2	2	-2	-2	2	0	0	2	-2	1	-1	-1	1
χ_{45}	1	1	3	3	2	2	2	0	0	0	-2	-2	-2	2	2	0	0	-2	-2	-1	-1	-1	-1
χ_{46}	1	-1	-3	3	2	-2	2	0	0	0	-2	2	-2	2	-2	0	0	-2	2	-1	1	1	-1
χ_{47}	1	-1	3	-3	2	-2	2	2	-2	2	2	-2	2	0	0	0	0	0	0	1	-1	1	-1
χ_{48}	1	-1	3	-3	2	-2	2	2	-2	2	2	-2	2	0	0	0	0	0	0	-1	1	-1	1
χ_{49}	1	1	-3	-3	2	2	2	-2	-2	-2	2	2	2	0	0	0	0	0	0	1	1	-1	-1
χ_{50}	1	1	-3	-3	2	2	2	-2	-2	-2	2	2	2	0	0	0	0	0	0	-1	-1	1	1
χ_{51}	0	0	0	0	-4	-4	-4	0	0	0	4	4	4	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	-4	4	-4	0	0	0	4	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{53}	1	3	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1
χ_{54}	1	-3	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1
χ_{55}	1	-3	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1
χ_{56}	1	3	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1
χ_{57}	0	0	0	0	1	1	-3	-3	1	1	1	1	-3	-1	1	-1	1	-1	1	0	0	0	0
χ_{58}	0	0	0	0	1	-1	-3	3	1	-1	1	-1	-3	-1	-1	1	1	-1	-1	0	0	0	0
χ_{59}	0	0	0	0	1	1	-3	-3	1	1	1	1	-3	1	-1	1	-1	1	-1	0	0	0	0
χ_{60}	0	0	0	0	1	-1	-3	3	1	-1	1	-1	-3	1	1	-1	-1	1	1	0	0	0	0
χ_{61}	0	0	0	0	-3	1	1	1	1	-3	-3	1	1	1	-1	1	-1	1	-1	0	0	0	0
χ_{62}	0	0	0	0	-3	1	1	1	1	-3	-3	1	1	-1	1	-1	1	-1	1	0	0	0	0
χ_{63}	0	0	0	0	-3	-1	1	-1	1	3	-3	-1	1	1	1	-1	-1	1	1	0	0	0	0
χ_{64}	0	0	0	0	-3	-1	1	-1	1	3	-3	-1	1	-1	-1	1	1	-1	-1	0	0	0	0
χ_{65}	-4	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Continue of Table 10

	6j	6k	12c	12d	5a	10a	10b	10c	2o	4o	4p	4q	4r	8d	4s	8e	3b	6l	6m	6n	12e
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1	1	-1	1	1	-1
χ_3	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	-1
χ_4	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	1
χ_5	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1
χ_6	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1
χ_7	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_8	0	0	0	0	0	0	0	0	-1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1
χ_9	1	-1	-1	1	0	0	0	0	3	-3	3	-3	-1	1	1	-1	2	-2	2	0	0
χ_{10}	-1	1	1	-1	0	0	0	0	-3	3	-3	3	-1	1	1	-1	2	-2	2	0	0
χ_{11}	-1	-1	-1	-1	0	0	0	0	3	3	3	3	-1	-1	-1	-1	2	2	2	0	0
χ_{12}	1	1	1	1	0	0	0	0	-3	-3	-3	-3	-1	-1	-1	-1	2	2	2	0	0
χ_{13}	-1	-1	-1	-1	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	1	-1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	1	1	1	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	-1	1	1	-1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	-1	-1	-1	-1	-3	-3	-3	-3	1	1	1	1	0	0	0	0	0
χ_{18}	0	0	0	0	-1	1	1	-1	-3	3	-3	3	1	-1	-1	1	0	0	0	0	0
χ_{19}	0	0	0	0	-1	-1	-1	-1	3	3	3	3	1	1	1	1	0	0	0	0	0
χ_{20}	0	0	0	0	-1	1	1	-1	3	-3	3	-3	1	-1	-1	1	0	0	0	0	0
χ_{21}	-1	1	1	-1	0	0	0	0	2	-2	2	-2	0	0	0	0	1	-1	1	-1	1
χ_{22}	1	-1	-1	1	0	0	0	0	-2	2	-2	2	0	0	0	0	1	-1	1	1	-1
χ_{23}	1	1	1	1	0	0	0	0	2	2	2	2	0	0	0	0	1	1	1	-1	-1
χ_{24}	-1	-1	-1	-1	0	0	0	0	-2	-2	-2	-2	0	0	0	0	1	1	1	1	1
χ_{25}	-1	1	-1	1	0	0	0	0	3	-1	-1	3	-1	1	-1	1	0	0	0	0	0
χ_{26}	1	1	-1	-1	0	0	0	0	3	1	-1	-3	-1	-1	1	1	0	0	0	0	0
χ_{27}	1	-1	1	-1	0	0	0	0	-3	1	1	-3	-1	1	-1	1	0	0	0	0	0
χ_{28}	-1	-1	1	1	0	0	0	0	-3	-1	1	3	-1	-1	1	1	0	0	0	0	0
χ_{29}	1	-1	1	-1	0	0	0	0	-3	1	1	-3	1	-1	1	-1	0	0	0	0	0
χ_{30}	-1	-1	1	1	0	0	0	0	-3	-1	1	3	1	1	-1	-1	0	0	0	0	0
χ_{31}	-1	1	-1	1	0	0	0	0	3	-1	-1	3	1	-1	1	-1	0	0	0	0	0
χ_{32}	1	1	-1	-1	0	0	0	0	3	1	-1	-3	1	1	-1	-1	0	0	0	0	0
χ_{33}	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	-2	-2	-2	0	0
χ_{34}	0	0	0	0	1	-1	-1	1	0	0	0	0	0	0	0	0	-2	2	-2	0	0
χ_{35}	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0

Continue of Table 10

	6j	6k	12c	12d	5a	10a	10b	10c	2o	4o	4p	4q	4r	8d	4s	8e	3b	6l	6m	6n	12e
χ_{36}	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0
χ_{37}	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0
χ_{38}	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0
χ_{39}	-1	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	1	-1	-1	1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	1	1	1	1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	-1	1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	-1	1	-1	1	0	0	0	0	-6	2	2	-6	0	0	0	0	0	0	0	0	0
χ_{48}	1	-1	1	-1	0	0	0	0	6	-2	-2	6	0	0	0	0	0	0	0	0	0
χ_{49}	1	1	-1	-1	0	0	0	0	-6	-2	2	6	0	0	0	0	0	0	0	0	0
χ_{50}	-1	-1	1	1	0	0	0	0	6	2	-2	-6	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0
χ_{54}	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0
χ_{55}	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0
χ_{56}	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0
χ_{57}	0	0	0	0	0	0	0	0	-3	1	1	-3	-1	1	-1	1	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	-3	-1	1	3	-1	-1	1	1	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	3	-1	-1	3	-1	1	-1	1	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	3	1	-1	-3	-1	-1	1	1	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	-3	1	1	-3	1	-1	1	-1	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	3	-1	-1	3	1	-1	1	-1	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	-3	-1	1	3	1	1	-1	-1	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	3	1	-1	-3	1	1	-1	-1	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0

5. Conclusion

The present character table for water clusters (cyclic forms) has been deduced from:
− the structure of group:

$$S_i[S_2] = (S_2 \times S_2 ... \times S_2) \wedge S_i^{'}. \text{ for } i = 2, 3, ...6$$

− The group of all feasible permutations is the wreath product $S_i[S_2]$ which consists of $i!(2)^i$ operations for $i = 2, ..., 6$ divided into (w.r.t) 5, 10, 20, 36, 65 conjugacy classes and 5, 10, 20, 36, 65 irreducible representations respectively.

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