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A Study of Some Geometric Aspects of a Subclass of Analytic Functions

Syed Zakar Hussain Bukhari[a*], Huo Tang[b], Farah Manzoor[a]

[a]Department of Mathematics, Mirpur University of Science and Technology(MUST), Mirpur-10250(AJK), Pakistan
[b]School of Mathematics and Computer Sciences, Chifeng University, Chifeng 024000, China

E-mail: fatmi@must.edu.pk
E-mail: thth2009@163.com
E-mail: fmhappiness@yahoo.com

Abstract. We introduce a subclass $k-TUS^{*}(\alpha,\vartheta)$of uniformly starlike functions $f$and study characterization theorem and coefficients estimates. We also define a neighbourhood of a function $f$under certain assumptions and study this neighborhood related results. We establish results relating to the partial sums of functions belonging to the class $k-TUS^{*}(\alpha,\vartheta)$. These functions are closely linked with the conformal mappings which lead to the growing applications in boundary and eigen-value problems in mathematics and various other fields of science and engineering. This research may also be related with the various known classes already found in the literature.

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*Corresponding Author

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1. Introduction

Let $\mathcal{A}$ denote the subclass of analytic function with the representation given below

$$\mathcal{A}=\left\{f: f(z)=z+\sum_{m=2}^{\infty} a_m z^m, \quad (z\in\mathbb{U}(0,1))\right\},$$

where $\mathbb{U}(0,1)=\{z\in\mathbb{C}: \mid z\mid<1\}$ is the unit disk in the $z-$plane with center at the origin and radius 1. A function f is univalent in a domain $\mathbb{U}(0,1)$, if it does not map the same point twice. A function f is univalent in $\mathbb{U}(0,1)$ provided that its derivative never vanishes and we prove the univalence of f by using Rouche's theorem. The notation $\mathcal{S}$ represents the class or family of univalent functions. The class $\mathcal{S}$ and its subclasses are related with the class $\mathcal{P}$ of functions p with $\Re(p(z))>0$ and

$$p(z)=1+\sum_{m=1}^{\infty} p_m z^m \quad (z\in\mathbb{U}(0,1)).$$

A large number of analytic functions classes are related with the class $\mathcal{P}$ and some of its generalizations. These include $\mathcal{S}^*\subset\mathcal{S}$, the class of starlike functions f satisfying the condition

$$\frac{zf'(z)}{f(z)}\in\mathcal{P}, \qquad (z\in\mathbb{U}(0,1)).$$

A function $f\in\mathcal{S}$ is convex in $\mathbb{U}(0,1)$, if and only if $zf'\in\mathcal{S}^*$. For these classes with order and arguments, see [17, 22, 23, 27] and others. These classes are further extended in such a way that the function f maps on to the right half plane or some specific plane region Δ_k defined by

$$\Delta_k=\left\{w=u+iu': u^2>k^2\left((u-1)^2+(u')^2\right), k\geq 0\right\},$$

under certain restriction on k, for detail see [8, 10, 11, 12, 16, 18]. The classes $k-\mathcal{US}^*(\alpha)$ and $k-\mathcal{UC}(\alpha)$ contain k-uniformly starlike and convex functions of order α respectively. We define these classes in the following.

Let $f\in\mathcal{A}$. Then $f\in k-\mathcal{US}^*(\alpha)$, if

$$\Re\left\{\frac{zf'(z)}{f(z)}-\alpha\right\}>k\left|\frac{zf'(z)}{f(z)}-1\right| \qquad (z\in\mathbb{U}(0,1)), \tag{1.1}$$

where $-1\leq\alpha<1$ and $k\geq 0$. Also $f\in k-\mathcal{UC}(\alpha)$ if and only if $zf'\in k-\mathcal{US}^*(\alpha)$. For $\alpha=0$ and $k=1$, these classes were studied by Goodman [4, 5] and Rönning [24, 25]. These and related subclasses are nicely extended in various aspects in the literature of the subject. Importantly Kanas and Srivastava also determined conditions on the parameters for which a certain linear operator maps the classes of starlike and univalent functions onto the class $k-\mathcal{UC}$ of uniformly convex and the class $k-\mathcal{US}^*$ of uniformly starlike functions, for reference, see [9]. Uniformly univalent functions with respect to symmetric points were also discussed in [19, 37, 38]. Goodman [6] in 1957 and Ruscheweyh

[26] in 1981, introduced and extended the concept of the neighbourhood of an analytic function f defined below:

$$\mathcal{N}_\tau(f)=\left\{h\in\mathcal{T}:h(z)=z-\sum_{m=2}^{\infty}b_m z^m,\ \sum_{m=2}^{\infty}m\mid a_m-b_m\mid\leq\tau\right\}.$$

Remark 1.1. For $I(z)=z$, we write

$$\mathcal{N}_\tau(I)=\left\{h\in\mathcal{T}:h=z-\sum_{m=2}^{\infty}b_m z^m,\ \sum_{m=2}^{\infty}m\mid b_m\mid\leq\tau\right\}. \tag{1.2}$$

Definition 1.2. Let $f,\ F\in\mathcal{A}$. Then f is subordinate to F ($f\prec F$), if for a "Schwarz function" w, $f(z)=F(w(z))$.

For $F\in\mathcal{S}$, $f\prec F\Leftrightarrow f(0)=F(0)$ and $f(\mathbb{U}(0,1))\subset F(\mathbb{U}(0,1))$. Let

$$\{f_n(z)\}=\left\{z-\sum_{m=2}^{n}a_m z^m,\qquad (z\in\mathbb{U}(0,1))\right\}$$

be a sequence of the partial sums of f. We choose the coefficients of f sufficiently small so that we can easily find the ratio of a function f to its sequence $f_n(z)=z-\sum_{m=2}^{n}a_m z^m$ and $f_1(z)=z$. We also determine the sharp lower bounds for these ratios. For this purpose, we have the following

$$\Re\left(\frac{1-\omega(z)}{1+\omega(z)}\right)>0\quad\text{if and only if}\quad\omega(z)=\sum_{m=1}^{\infty}d_m z^m\ \text{ with }\ |\omega(z)|\leq|z|.$$

A function $f\in\mathcal{T}$ can be written as

$$f(z)=z-\sum_{m=2}^{\infty}a_m z^m,\qquad (a_m\geq 0). \tag{1.3}$$

Silverman [32] introduced some subclasses of $\mathcal{T}$. These and related classes were further investigated in [13, 14, 20, 21, 33, 39] and others. We now introduce the following:

Definition 1.3. A function $f\in k-\mathcal{US}^*(\alpha,\vartheta)$, if

$$\Re\left(\vartheta\frac{z^2f''(z)}{f(z)}+(1+ke^{i\theta})\frac{zf'(z)}{f(z)}-ke^{i\theta}\right)>\alpha, \tag{1.4}$$

for $z\in\mathbb{U}(0,1)\,,\vartheta\geq 0, 0\leq\alpha<1, 0<k<1$.

To define the subclass $k-\mathcal{TUS}^*(\alpha,\vartheta)$ of analytic functions with non-negative coefficients, we take

$$k-\mathcal{TUS}^*(\alpha,\vartheta)=k-\mathcal{US}^*(\alpha,\vartheta)\cap\mathcal{T}. \tag{1.5}$$

The classes of functions related with these classes are studied in [1, 2, 3, 10, 11, 12, 18, 25, 36] and others.

2. Some basic lemmas

From [28], we have the following two lemmas.

Lemma 2.1. *If $w \in \mathbb{C}$ and $\vartheta \in \mathbb{R}$, then we have*

$$\Re(w(z)) \geq \vartheta \Leftrightarrow | w(z) + (1-\vartheta) | - | w(z) - (1-\vartheta) | \geq 0.$$

The above lemma relates the functions with positive real part to the functions defined by using the absolute values

Lemma 2.2. *For $w \in \mathbb{C}$ and $\vartheta \in \mathbb{R}$, we can write*

$$\Re(w(z)) \geq k \mid \vartheta + w(z) - 1 \mid \Longleftrightarrow \Re\{w(z)(1+ke^{i\theta}) - ke^{i\theta}\} \geq \vartheta, \quad -\pi \leq \theta \leq \pi.$$

This lemma also relates the functions having some plane curves as image domains to the functions defined by using the positive real part. The concept of $\prec$ leads to the following lemma:

Lemma 2.3. *If $f, h \in \mathcal{A}$ with $f \prec h$, then*

$$\int_0^{2\pi} \left| f(re^{i\theta}) \right|^{\tau} d\theta \leq \int_0^{2\pi} \left| h(re^{i\theta}) \right|^{\tau} d\theta, \tag{2.1}$$

where $z = re^{i\theta}$, $0 < r < 1$ and $\tau > 0$.

For the reference of the above result, see [15].

3. Main results

In 1957, Goodman in [15] studied certain coefficient conditions for the convexity and starlikeness of some subclasses of analytic functions. These results were further extended in the prospective of various other related subclasses of analytic, harmonic and meromorphic functions. In the following, we study the characterizations of functions in the class $k - \mathcal{TUS}^*(\alpha, \vartheta)$ under certain restrictions on the parameters involved.

Theorem 3.1. *A function $f \in k - \mathcal{TUS}^*(\alpha, \vartheta)$ iff*

$$\sum_{m=2}^{\infty} [(m-1)(k+1)(\vartheta m + 1) + (1-\alpha)] a_m \leq 1 - \alpha. \tag{3.1}$$

The above inequality is sharp.

Proof. Suppose (3.1) holds. Then, we can write

$$\begin{aligned} |F(z) - 1| &= \left| \frac{(\vartheta z^2 f''(z) + zf'(z))(1+ke^{i\theta}) - ke^{i\theta} f(z)}{f(z)} - 1 \right| \\ &= \left| \frac{\sum_{m=2}^{\infty} [\vartheta k e^{i\theta} m(m-1) + \vartheta m(m-1) + m + ke^{i\theta} m - ke^{i\theta} - 1] a_m z^{m-1}}{1 - \sum_{m=2}^{\infty} a_m z^{m-1}} \right| \\ &\leq \left| \frac{\sum_{m=2}^{\infty} [(m-1)(k+1)(\vartheta m + 1)] a_m}{1 - \sum_{m=2}^{\infty} a_m} \right| \leq 1 - \alpha. \end{aligned}$$

Thus $|F(z)-1| \leq 1-\alpha$, that is

$$\frac{(\vartheta z^2 f''(z)+zf'(z))(1+ke^{i\theta})-ke^{i\theta}f(z)}{f(z)}$$

lies in the disk centered at 1 and radius $1-\alpha$. Thus, $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$.

Conversely, suppose that $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$. Then from (1.4), we have

$$\Re\left(\frac{1-\sum_{m=2}^{\infty}[\vartheta m(m-1)+m+\vartheta ke^{i\theta}m(m-1)+ke^{i\theta}m-ke^{i\theta}f(z)]a_m z^{m-1}}{1-\sum_{m=2}^{\infty}a_m z^{m-1}}\right)>\alpha.$$

For real values we take $z \longrightarrow 1^-$ and write

$$\sum_{m=2}^{\infty}[(m-1)(k+1)(\vartheta m+1)+1-\alpha]a_m \leq 1-\alpha$$

as required. The following function

$$f(z)=z-\frac{1-\alpha}{(m-1)(k+1)(\vartheta m+1)+1-\alpha}z^m, \quad (m \geq 2) \tag{3.2}$$

provides the sharpness for the assertion (3.1). □

Corollary 3.2. *Let $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$. Then for $m \geq 2$, we have*

$$a_m \leq \frac{1-\alpha}{(1+k)(1+\vartheta m)(m-1)+1-\alpha}. \tag{3.3}$$

The following Theorem deals with the inclusion for $\vartheta_1 < \vartheta_2$.

Theorem 3.3. *Let $0 \leq \alpha < 1, 0 \leq \vartheta_1 < \vartheta_2$ and $0 < k < 1$. Then $k-\mathcal{TUS}^*(\alpha,\vartheta_2) \subset k-\mathcal{TUS}^*(\alpha,\vartheta_1)$.*

Proof. For $f \in k-\mathcal{TUS}^*(\alpha,\vartheta_2)$, we write

$$\sum_{m=2}^{\infty}[(m-1)(k+1)(1+m\vartheta_1)+(1-\alpha)]a_m < \sum_{m=2}^{\infty}[(m-1)(k+1)(m\vartheta_2+1)+(1-\alpha)]a_m \leq 1-\alpha.$$

Hence $f \in k-\mathcal{TUS}^*(\alpha,\vartheta_1)$. □

From the above theorem, we deduce that $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$ is $k-$uniformly starlike and starlike of order α.

Following the results of Goodman [6] and Ruscheweyh [26], we find the neighbourhood of $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$.

Theorem 3.4. *Let $0 \leq \alpha < 1, 0 \leq \vartheta_1 < \vartheta_2$ and $0 < k < 1$. Then $k-\mathcal{TUS}^*(\alpha,\vartheta) \subseteq \mathcal{N}_\tau(I)$, where $I(z)=z$ and*

$$\tau=\frac{2(1-\alpha)}{(2\vartheta+1)(k+1)+(1-\alpha)}.$$

Proof. For $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$ and by Theorem 3.1, we have

$$[(2\vartheta+1)(k+1)+(1-\alpha)]\sum_{m=2}^{\infty}a_m \leq \sum_{m=2}^{\infty}[(1+\vartheta m)(1+k)(m-1)+(1-\alpha)]a_m \leq 1-\alpha,$$

which yields

$$\sum_{m=2}^{\infty} a_m \leq \frac{1-\alpha}{(k+1)(2\vartheta+1)+(1-\alpha)}. \tag{3.4}$$

Also, we have

$$(2\vartheta+1)(k+1)\sum_{m=2}^{\infty} m a_m \leq (1-\alpha) - [(1-\alpha)-(2\vartheta+1)(k+1)]\sum_{m=2}^{\infty} a_m.$$

Using (3.4), we can write

$$(1+k)(2\vartheta+1)\sum_{m=2}^{\infty} m a_m \leq 1-\alpha - \frac{(1-\alpha)\left[1-\alpha-(1+k)(2\vartheta+1)\right]}{(2\vartheta+1)(k+1)+(1-\alpha)}.$$

On simplifying the above inequality, we see that

$$\sum_{m=2}^{\infty} m a_m \leq \frac{2(1-\alpha)}{(2\vartheta+1)(k+1)+(1-\alpha)} = \tau,$$

which, in view of (1.2), leads the proof. □

Letting $\vartheta = 0$, we have $k-\mathcal{TUS}^*(\alpha) \subseteq \mathcal{N}_\tau(e)$, where $\tau = \frac{2(1-\alpha)}{k-\alpha+2}$ and $k-\mathcal{TUS}^*(\alpha)$ is a subclass of $k-$uniformly starlike analytic functions with non-negative coefficient of order α. Also, if we take $\vartheta = k = 0$, then we obtain that $\mathcal{TUS}^*(\alpha) \subseteq \mathcal{N}_\tau(e)$, where $\tau = \frac{2(1-\alpha)}{\alpha+2}$ and $\mathcal{TUS}^*(\alpha)$ is a subclass of starlike analytic functions with non-negative coefficient of order α. Silverman [32] found that $f_1(z) = z - \frac{z^2}{2}$ is an extremal for $\mathcal{T}$. He used it to solve the integral inequality conjectured in [29] and finally settled in [30], that is

$$\int_0^{2\pi} |f(z)|^\eta \, d\theta \leq \int_0^{2\pi} |f_1(z)|^\eta \, d\theta, \;\; f_1 \in \mathcal{T}\;, \;\; 0<r<1.$$

We solve the conjecture for $k-\mathcal{TUS}^*(\alpha,\vartheta)$. We use the concept of subordination already defined above and a known result of Littlewood seen in [15].

Theorem 3.5. *If $f \in k-\mathcal{TUS}^*(\alpha,\vartheta)$ and $z = re^{i\theta}$, $r<1$ with $\tau>0$, then*

$$\int_0^{2\pi} \left|f\left(re^{i\theta}\right)\right|^\tau d\theta \leq \int_0^{2\pi} \left|f_1\left(re^{i\theta}\right)\right|^\tau d\theta,$$

where

$$f_1(z) = z - \frac{(1-\alpha)z^2}{2\vartheta k + 2\vartheta + k + 2 - \alpha}. \tag{3.5}$$

Proof. Let f and f_1 be given by equations (1.3) and (3.5) respectively. Then, we write

$$\int_0^{2\pi} \left|1 - \sum_{m=2}^{\infty} a_m z^{m-1}\right|^\tau d\theta \leq \int_0^{2\pi} \left|1 - \frac{1-\alpha}{2\vartheta k + 2\vartheta + k + 2 - \alpha} z\right|^\tau d\theta.$$

By using Lemma 2.2, we have

$$1-\sum_{m=2}^{\infty} a_m z^{m-1} \prec 1-\frac{1-\alpha}{2\vartheta k+2\vartheta+k+2-\alpha} z,$$

or we can write

$$1-\sum_{m=2}^{\infty} a_m z^{m-1}=1-\frac{1-\alpha}{2\vartheta k+2\vartheta+k+2-\alpha} \omega(z). \tag{3.6}$$

From (3.1) and (3.6), we obtain

$$|\omega(z)| \leq |z| \sum_{m=2}^{\infty} \frac{[(m-1)(k+1)(\vartheta m+1)+1-\alpha]}{1-\alpha} a_m \leq |z| .$$

This completes the proof. □

For $f \in k-\mathcal{TUS}^*(\alpha)$ with $\tau>0$, we have

$$\int_0^{2\pi}\left|f\left(re^{i\theta}\right)\right|^\tau d\theta \leq \int_0^{2\pi}\left|f_1\left(re^{i\theta}\right)\right|^\tau d\theta,$$

where $f_1(z)=z-\frac{(1-\alpha)}{k+2-\alpha} z^2$. Also for $f \in \mathcal{TUS}^*(\alpha)$ with $\tau>0$, we deduce

$$\int_0^{2\pi}\left|f\left(re^{i\theta}\right)\right|^\tau d\theta \leq \int_0^{2\pi}\left|f_1(re^{i\theta})\right|^\tau d\theta,$$

where $f_1(z)=z-\frac{1-\alpha}{2-\alpha} z^2$.

Theorem 3.6. *If $f \in k-\mathcal{TUS}^*(\alpha, \vartheta)$, then*

$$\Re \frac{f\ (z)}{f_n(z)} \geq \frac{d_{n+1}-1}{d_{n+1}} \tag{3.7}$$

and

$$\Re \frac{f_n(z)}{f\ (z)} \geq \frac{d_{n+1}}{1+d_{n+1}}, \tag{3.8}$$

where

$$d_m=: \frac{[(m-1)(k+1)(\vartheta m+1)+1-\alpha]}{1-\alpha}.$$

These estimates in (3.7) *and* (3.8) *are sharp.*

Proof. Using the similar procedure as adopted by Silverman [31], we see that $f \in k-\mathcal{TUS}^*(\alpha, \vartheta)$ if and only if $\sum_{m=2}^{\infty} dma_m \leq 1$. Clearly, $d_{m+1}>d_m>1$. So we can write

$$\sum_{m=2}^{n} a_m+d_{n+1} \sum_{m=n+1}^{\infty} a_m \leq \sum_{m=2}^{\infty} d_m a_m \leq 1, \tag{3.9}$$

where $d_m =: \frac{[(m-1)(k+1)(\vartheta m+1)+1-\alpha]}{1-\alpha}$. We also have

$$\frac{[n(k+1)(\vartheta\,(n+1)+1)+1-\alpha]}{1-\alpha}\frac{f(z)}{f_n(z)}-\frac{n(k+1)(\vartheta\,(n+1)+1)+1-\alpha}{1-\alpha}+1$$
$$=\frac{2-2\alpha-(1-\alpha)\sum_{m=2}^{n}a_m z^{m-1}-[n(k+1)(\vartheta n+\vartheta+1)+1-\alpha]\sum_{m=n+1}^{\infty}a_m z^{m-1}}{1-\sum_{m=2}^{k}a_m z^{m-1}}$$
$$=\frac{1+C(z)}{1+D(z)}.$$

From the assertion $\frac{1+C(z)}{1+D(z)}=\frac{1-\omega(z)}{1+\omega(z)}$, we have $\omega(z)=\frac{D(z)-C(z)}{2+C(z)+D(z)}$. Thus

$$|\omega(z)|\leq\frac{\{n(k+1)(\vartheta n+\vartheta+1)+(1-\alpha)\}\sum_{m=n+1}^{\infty}a_m}{2-2\alpha-2\,(1-\alpha)\sum_{m=2}^{n}a_m-\{n(k+1)(\vartheta n+\vartheta+1)+1-\alpha\}\sum_{m=n+1}^{\infty}a_m}.$$

Now $|\omega(z)|\leq 1$ if and only if

$$\sum_{m=2}^{n}a_m+\frac{[n(k+1)(\vartheta\,(n+1)+1)+1-\alpha]}{1-\alpha}\sum_{m=n+1}^{\infty}a_m\leq 1, \tag{3.10}$$

which is true by (3.9) and we get the assertion (3.7). To check the sharpness, we consider

$$f(z)=z-\frac{(1-\alpha)\,z^{n+1}}{n(k+1)(\vartheta\,(n+1)+1)+1-\alpha}\ ,\quad \frac{f(z)}{f_n(z)}=1-\frac{(1-\alpha)\,z^{n}}{n(k+1)(\vartheta\,(n+1)+1)+1-\alpha}, \tag{3.11}$$

and letting $z\to 1^-$, we write

$$\frac{f\ (z)}{f_n(z)}=1-\frac{1-\alpha}{[n(k+1)(\vartheta\,(n+1)+1)+(1-\alpha)]},$$

which shows that (3.7) is the best possible for $n\in\mathbb{N}$. Now consider that

$$(1+d_{n+1})\,\frac{f_n(z)}{f(z)}-d_{n+1}=\frac{1-\sum_{m=2}^{n}a_m z^{m-1}+\frac{n(k+1)(\vartheta n+\vartheta+1)+(1-\alpha)}{1-\alpha}\sum_{m=n+1}^{\infty}a_m z^{m-1}}{1-\sum_{m=2}^{n}a_m z^{m-1}}$$
$$=\frac{1-\omega(z)}{1+\omega(z)}.$$

where

$$d_{n+1}=\frac{[n(k+1)(\vartheta\,(n+1)+1)+(1-\alpha)]}{1-\alpha}.$$

Then we have

$$|\omega(z)|\leq\frac{\sum_{m=n+1}^{\infty}\{2-2\alpha+n(k+1)(\vartheta n+\vartheta+1)\}\,a_m}{2-2\alpha-2\,(1-\alpha)\sum_{m=2}^{n}a_m+\sum_{m=n+1}^{\infty}\{2-2\alpha-n(k+1)(\vartheta n+\vartheta+1)\}\,a_m}.$$

Using (3.10), we obtain (3.8). These estimates in (3.7) and (3.8) are sharp for f in (3.11). □

We note that $k-\mathcal{TUS}^*(\alpha,\vartheta)=k-\mathcal{TUS}^*(\alpha)$ and $0-\mathcal{TUS}^*(\alpha,0)=\mathcal{TUS}^*(\alpha)$. In the following, we find the ratios of the derivatives.

Theorem 3.7. *If $f\in k-\mathcal{TUS}^*(\alpha,\vartheta)$, then*

$$\Re\left[\frac{f'(z)}{f'_n(z)}\right]\geq\frac{1}{d_{n+1}}(d_{n+1}-n-1)\quad\textit{and}\quad\Re\left[\frac{f'_n(z)}{f'(z)}\right]\geq d_{n+1}\left(\frac{1}{d_{n+1}+n+1}\right),$$

where

$$d_m =: \frac{[(m-1)(k+1)(\vartheta m+1)+(1-\alpha)]}{1-\alpha}.$$

The lower bounds are sharp for f given by (3.11).

Following procedure of Theorem 3.6, we can easily prove this Theorem.

4. Summary and concluding remarks

In this research, we studied a subclass $k-\mathcal{TUS}^*(\alpha,\vartheta)$ of uniformly starlike functions including characterization theorem and coefficients estimates. Also we defined a neighbourhood of an analytic function f under certain assumptions and studied some neighborhood related results. We established results relating to the partial sums of functions in the class $k-\mathcal{TUS}^*(\alpha,\vartheta)$. This research can be related with the various known subclasses of analytic functions already seen in the literature and it may be kept updated with the emerging trends in geometric functions theory as found in [34, 35].

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