Iranian Journal of Mathematical Sciences and Informatics
Vol. 18, No. 2 (2023), pp 107-115
DOI: 10.52547/ijmsi.18.2.107

# Cordial Labeling of Corona Product between Paths and Fourth Power of Paths

S. Nada[a], A. Elrokh[a], A. Elrayes[b], A. Rabie[b*]

[a]Department of Mathematics, Faculty of Science, Menoyfia University, Egypt
[b]Institute of National Planning, Cairo, Egypt

E-mail: shokrynada@yahoo.com
E-mail: el-rokh@excite.com
E-mail: amani.elrayes@inp.edu.eg
E-mail: aya.ebrahim828@yahoo.com

Abstract. A simple graph is said to be cordial if it admits 0-1 labeling that satisfies certain conditions. In this paper we investigate necessary and sufficient conditions for cordial labeling of the corona product between paths and fourth power of paths be cordial.

**Keywords:** Path, Corona, Cordial labeling, Fourth power.

**2000 Mathematics subject classification:** 05C78, 05C75, 05C20.

## 1. Introduction

Let G be a graph with $p$ vertices and $q$ edges. All graphs considered here are simple, finite, connected and undirected. A labeling of a graph G is a process of allocating numbers or labels to the nodes of $G$ or lines of $G$ or both through mathematical functions [2]. There are several types of labeling and a complete survey of graph labeling is available in [6]. Graph theory has a good development in the graph labeling and has a broad range of applications, some of which were reported in the work of Yegnanaryanan and Vaidhyanathan

*Corresponding Author

Received 20 April 2019; Accepted 26 March 2020

[9]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by Cahit in [3]. Extensions of this labeling include mean cordial labeling, $H_1$- and $H_2$-cordial labeling of some graphs [7]. In 1990, Cahit [4], proved the following: each tree is cordial; an Euerlian graph is not cordial if its size is congruent to $2(mod\ 4)$ ; a complete graph $K_n$ is cordial if and only if $n \leq 3$ and a complete bipartite graph $K_{n,m}$ is cordial for all positive integers $n$ and $m$. Let $G_1$, $G_2$ respectively be $(p_1, q_1)$, $(p_2, q_2)$ graphs. The corona $G_1 \bigodot G_2$ of two graphs $G_1$ (with $n_1$ vertices , $m_1$ edges) and $G_2$ (with $n_2$ vertices , $m_2$ edges) is defined as the graph obtained by taking one copy of $G_1$ and copies of $G_2$ , and then joining the $i^{th}$ vertex of $G_1$ with an edge to every vertex in the $i^{th}$ copy of $G_2$ . It is easy to see that the corona $G_1 \bigodot G_2$ that has $n_1 + n_1n_2$ vertices and $m_1 + n_1m_2 + n_1n_2$ edges. We will give a brief summary of definitions which are useful for the present investigations.

**Definition 1.1.** A mapping $f{:}V \rightarrow \{0, 1\}$ is called *binary vertex labeling* of $G$ and $f(v)$ is called *the label of the vertex v of G under f.* For any edge $e{=}uv$ , the induced edge labeling $f^*{:}E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$, where $u, v \in V$. Let $v_f(i)$ be the numbers of vertices of $G$ labeled $i$ under $f$ , and $e_f(i)$ be the numbers of edges of $G$ labeled $i$ under $f^*$ where $i \in \{0, 1\}$.

**Definition 1.2.** A binary vertex labeling of a graph $G$ is called *cordial* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if it admits a cordial labeling.

**Definition 1.3.** The fourth power of a cycles $C_n$ denoted by $C_n^4$ , is $C_n \bigcup J$, where J is the set of all edges of the form edges $v_iv_j$ such that $2 \leq d(v_iv_j) \leq 4$, where $d(v_iv_j)$ is the shortest distance from $v_i$ to $v_j$.

## 2. Terminology and notation

A path with $m$ vertices and $m-1$ edges, denoted by $P_m$, and its fourth power $P_n^4$ has $n$ vertices and $4n - 10$ edges. We let $L_{4r}$ denote the labeling 0011 0011...0011 "$r$-times", Let $L'_{4r}$ denote the labeling 0110 0110...0110 "$r$-times". The labeling 1100 1100...1100 "$r$-times" and labeling 1001 1001...1001 "$r$-times" are written $S_{4r}$ and $S'_{4r}$. Let $M_{2r}$ denote the labeling 0101...01, zero-one "$r$times". We let $M'_{2r}$ denote the labeling 1010...10. Regularly, we modify the labeling $M_2r$ or $M'_2r$ by adding symbols at one end or the other (or both). Also, $L_{4r}$ (or $L'_{4r}$ ) with extra labeling from right or left (or both sides). Let us use $\alpha_i$ to indicate the labeling of $P_n^4$ that is adjacent to a vertex of $P_m$ that is labeled i, $i = 0, 1$ of the corona $P_m \bigodot P_n^4$. Use $y_i$ , $b_i$ $(i = 0, 1)$ to denote the number of vertices and edges, respectively for $\alpha_0$ of $P_n^4$, and consider $y'_i$, $b'_i$ $(i = 0, 1)$ to denote the number of vertices and edges, respectively for $\alpha_1$ of $P_n^4$. Sometimes, we use the notation $\alpha{*}_0$ for the labeling of $P_n^4$ which is only associated to the last vertex labeled 0 of $P_m$. In this case, we will use the



[ DOI: 10.61186/ijmsi.18.2.107 ]

notation $b_0^{'*}$, $b_1^{'*}$, $y_0^{'*}$ and $y_1^{'*}$ instead of $b_0'$, $b_1'$, $y_0'$ and $y_1'$, respectively. Similarly, the notation $\alpha *_1$ may be used for the labeling of $P_n^4$ that is associated only to the last vertex labeled 1 of $P_m$. It is easy to verify that $v_0 = x_0 + x_0 y_0 + x_1 y_0'$, $v_1 = x_1 + x_0 y_1 + x_1 y_1'$, $e_0 = a_0 + x_0 b_0 + x_1 b_0' + x_0 y_0 + x_1 y_1'$ and $e_1 = a_1 + x_0 b_1 + x_1 b_1' + x_0(x_0 y_1) + x_1 y_0'$. Thus, $v_0 - v_1 = (x_0 - x_1) + x_0(y_0 - y_1) + x_1(y_0' - y_1')$ and $e_0 - e_1 = (a_0 - a_1) + x_0(b_0 - b_1) + x_1(b_0' - b_1') + x_0(y_0 - y_1) - x_1(y_0' - y_1')$. When it comes to the proof, we only need to show that, for each specified combination of labeling, $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$.

## 3. Main results

In this section, we study cordial labeling of the corona product between paths and fourth power of paths and show that all $P_m \bigodot P_n^4$ are cordial for all integers $m \geq 1$ when $n \geq 7$, and for all integers $m > 1$ when $n = 3$.

**Lemma 3.1.** *The corona $P_m \bigodot P_3^4$ is cordial if and only if $m \neq 1$.*

*Proof.* Since $P_3^4 = C_3$, $P_m \bigodot P_3^4$ is cordial [8]. □

**Lemma 3.2.** *If $n \equiv 0(mod\ 4)$, $n \geq 8$, then $P_m \bigodot P_n^4$ is cordial for all $m \geq 1$.*

*Proof.* Suppose that $n = 4s$, where $s \geq 2$. The following cases will be examined.
**Case 1.**
Suppose that $m = 1$. Choose the labeling 0 for $P_1$ and the labeling $\alpha_0 = 0L_{4s-4}01_2$ for $P_{4s}^4$. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5$ and obviously $y_0' = y_1' = b_0' = b_1' = 0$ . It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_1 \bigodot P_{4s}^4$ is cordial. As an example, Figure (1) illustrates $P_1 \bigodot P_8^4$.

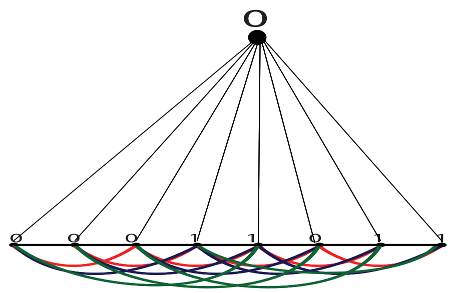


Figure 1

**Case 2.**



[ DOI: 10.61186/ijmsi.18.2.107 ]

Suppose that $m = 2$. Choose the labeling 01 for $P_2$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_2 \bigodot P^4_{4s}$ is cordial.

**Case 3.**

Suppose that $m = 3$. Choose the labeling 001 for $P_3$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2, x_1 = 2, a_0 = a_1 = 1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_3 \bigodot P^4_{4s}$ is cordial.

**Case 4.** $m \equiv 0(mod\ 4)$.

Suppose that $m = 4r$, $r \geq 2$. Choose the labeling $L_{4r}$ for $P_{4r}$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 2r, a_0 = 2r, a_1 = 2r-1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \bigodot P^4_{4s}$ is cordial.

**Case 5.** $m \equiv 1(mod\ 4)$.

Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $L_{4r}0$ for $P_{4r+1}$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = a_1 = 2r, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+1} \bigodot P^4_{4s}$ is cordial.

**Case 6.** $m \equiv 2(mod\ 4)$.

Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling $L_{4r}01$ for $P_{4r+2}$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = x_1 = 2r+1, a_0 = 2r, a_1 = 2r+1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+2} \bigodot P^4_{4s}$ is cordial.

**Case 7.** $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $L_{4r}001$ for $P_{4r+3}$. Take $\alpha_0$ to be $0L_{4s-4}01_2$ and $\alpha_1$ to be $1_2L'_{4s-4}0_2$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = a_1 = 2r+1, y_0 = y_1 = 2s, b_0 = b_1 = 8s-5, y'_0 = y'_1 = 2s$ and $b'_0 = b'_1 = 8s-5$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+3} \bigodot P^4_{4s}$ is cordial. □

**Lemma 3.3.** *If $n \equiv 1(mod\ 4)$, then $P_m \bigodot P^4_n$ is cordial for all $m \geq 1$.*

*Proof.* Suppose that $n = 4s+1$, where $s \geq 2$. The following cases will be examined.

**Case 1.**

Suppose that $m = 1$. Choose the labeling 0 for $P_1$ and the labeling $\alpha_0 = 1_2L'_{4s-4}010$ for $P^4_{4s+1}$. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = 2s, y_1 = 2s+1, b_0 = b_1 = 8s-3$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$ . It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_1 \bigodot P^4_{4s+1}$ is cordial.

**Case 2.**

Suppose that $m = 2$. Choose the labeling 01 for $P_2$. Take $\alpha_0$ to be $0_2L_{4s-4}101$ and $\alpha_1$ to be $1_2L'_{4s-4}010$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 =$



[ DOI: 10.61186/ijmsi.18.2.107 ]

$2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s+1, y'_1 = 2s$ and $b'_0 = b'_1 = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. As an example, Figure (2) illustrates $P_2 \bigodot P_9^4$. Hence, $P_2 \bigodot P_{4s+1}^4$ is cordial.

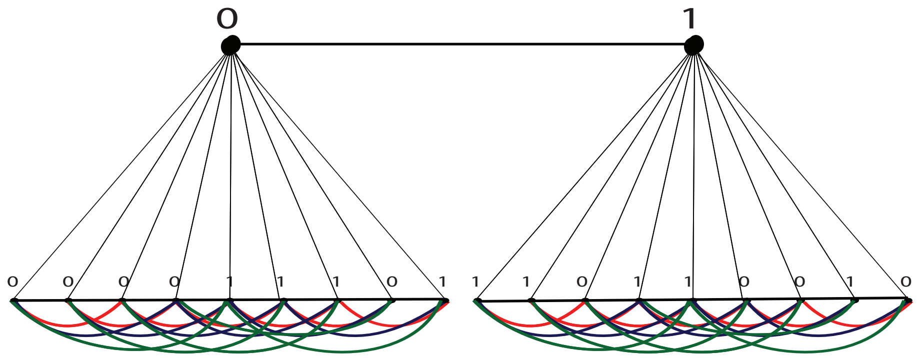


FIGURE 2

**Case 3**.
Suppose that $m = 3$. Choose the labeling 010 for $P_3$. Take $\alpha_0$ (associated to the first vertex labeled 0 in $P_3$) to be $0_2L_{4s-4}101$, $\alpha_1$ to be $1_2L'_{4s-4}010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_3$) to be $1_2L'_{4s-4}010$. Therefore $x_0 = 2, x_1 = 1, a_0 = 0, a_1 = 2, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s, y'_1 = 2s+1, b'_0 = b'_1 = 8s-3, y_0^{'*} = 2s, y_1^{'*} = 2s+1$ and $b_0^{'*} = b_1^{'*} = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_3 \bigodot P_{4s+1}^4$ is cordial.
**Case 4**. $m \equiv 0(mod\ 4)$.
Suppose that $m = 4r$, $r \geq 1$. Choose the labeling $M_{4r}$ for $P_{4r}$. Take $\alpha_0$ to be $0_2L_{4s-4}101$ and $\alpha_1$ to be $1_2L'_{4s-4}010$. Therefore $x_0 = x_1 = 2r, a_0 = 0, a_1 = 4r-1, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s, y'_1 = 2s+1$ and $b'_0 = b_1 = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \bigodot P_{4s+1}^4$ is cordial.
**Case 5**. $m \equiv 1(mod\ 4)$.
Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $M_{4r}0$ for $P_{4r+1}$. Take $\alpha_0$ to be $0_2L_{4s-4}101$, $\alpha_1$ to be $1_2L'_{4s-4}010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_{4r+1}$) to be $1_2L'_{4s-4}010$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = 0, a_1 = 4r, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s, y'_1 = 2s+1, b'_0 = b'_1 = 8s-3, y_0^{'*} = 2s, y_1^{'*} = 2s+1$ and $b_0^{'*} = b_1^{'*} = 8s-3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+1} \bigodot P_{4s+1}^4$ is cordial.
**Case 6**. $m \equiv 2(mod\ 4)$.
Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling $M_{4r+2}$ for $P_{4r+2}$. Take $\alpha_0$ to be $0_2L_{4s-4}101$ and $\alpha_1$ to be $1_2L'_{4s-4}010$. Therefore $x_0 = x_1 = 2r+1, a_0 = 0, a_1 = 4r+1, y_0 = 2s+1, y_1 = 2s, b_0 = b_1 = 8s-3, y'_0 = 2s, y'_1 = 2s+1$


[ DOI: 10.61186/ijmsi.18.2.107 ]

and $b'_0 = b'_1 = 8s - 3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence $P_{4r+2} \bigodot P^4_{4s+1}$ is cordial.

**Case 7.** $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r + 3$, $r \geq 1$. Choose the labeling $M_{4r+2}0$ for $P_{4r+3}$. Take $\alpha_0$ to be $0_2L_{4s-4}101$, $\alpha_1$ to be $1_2L'_{4s-4}010$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_{4r+3}$ to be $1_2L'_{4s-4}010$. Therefore $x_0 = 2r + 2, x_1 = 2r + 1, a_0 = 0, a_1 = 4r + 2, y_0 = 2s + 1, y_1 = 2s, b_0 = b_1 = 8s - 3, y'_0 = 2s, y'_1 = 2s + 1, b'_0 = b'_1 = 8s - 3, y_0^{'*} = 2s, y_1^{'*} = 2s + 1$ and $b_0^{'*} = b_1^{'*} = 8s - 3$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+3} \bigodot P^4_{4s+1}$ is cordial. □

**Lemma 3.4.** *If $n \equiv 2(mod\ 4)$, then $P_m \bigodot P^4_n$ is cordial for all $m \geq 1$.*

*Proof.* Suppose that $n = 4s + 2$, where $s \geq 2$. The following cases will be studied.

**Case 1.**

Suppose that $m = 1$. Choose the labeling 0 for $P_1$ and the labeling $\alpha_0 = 01_30S_{4s-4}0$ for $P^4_{4s+2}$. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = y_1 = 2s + 1, b_0 = b_1 = 8s - 1$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_1 \bigodot P^4_{4s+2}$ is cordial.

**Case 2.**

Suppose that $m = 2$. Choose the labeling 01 for $P_2$. Take $\alpha_0$ to be $01_30S_{4s-4}0$ and $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = y_1 = 2s + 1, b_0 = b_1 = 8s - 1, y'_0 = y'_1 = 2s + 1$ and $b'_0 = b'_1 = 8s - 1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_2 \bigodot P^4_{4s+2}$ is cordial.

**Case 3.**

Suppose that $m = 3$. Choose the labeling 001 for $P_3$. Take $\alpha_0$ to be $01_30S_{4s-4}0$ and $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = 2, x_1 = 1, a_0 = a_1 = 1, y_0 = y_1 = 2s + 1, b_0 = b_1 = 8s - 1, y'_0 = y'_1 = 2s + 1$ and $b'_0 = b'_1 = 8s - 1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. As an example, Figure (3) illustrates $P_3 \bigodot P^4_{10}$. Hence, $P_3 \bigodot P^4_{4s+2}$ is cordial.

**Case 4.** $m \equiv 0(mod\ 4)$.

Suppose that $m = 4r$, $r \geq 1$. Choose the labeling $L_{4r}$ for $P_{4r}$. Take $\alpha_0$ to be $01_30S_{4s-4}0$ and $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 2r, a_0 = 2r, a_1 = 2r - 1, y_0 = y_1 = 2s + 1, b_0 = b_1 = 8s - 1, y'_0 = y'_1 = 2s + 1$ and $b'_0 = b_1 = 8s - 1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r} \bigodot P^4_{4s+2}$ is cordial.

**Case 5.** $m \equiv 1(mod\ 4)$.

Suppose that $m = 4r + 1$, $r \geq 1$. Choose the labeling $L_{4r}0$ for $P_{4r+1}$. Take $\alpha_0$ to be $01_30S_{4s-4}0$, $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = 2r + 1, x_1 = 2r, a_0 = a_1 = 2r, y_0 = y_1 = 2s + 1, b_0 = b_1 = 8s - 1, y'_0 = y'_1 = 2s + 1$ and $b'_0 = b'_1 = 8s - 1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+1} \bigodot P^4_{4s+2}$ is cordial.

**Case 6.** $m \equiv 2(mod\ 4)$.

Suppose that $m = 4r + 2$, $r \geq 1$. Choose the labeling $L_{4r}01$ for $P_{4r+2}$ and


[ DOI: 10.61186/ijmsi.18.2.107 ]

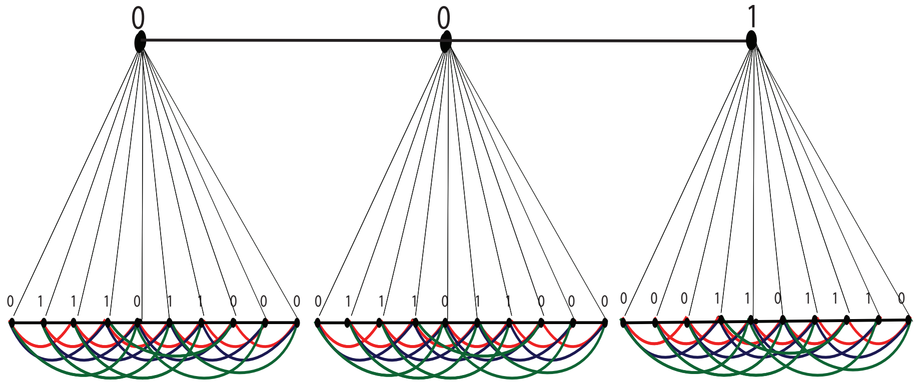


FIGURE 3

take $\alpha_0$ to be $01_30S_{4s-4}0$ and $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = x_1 = 2r+1, a_0 = 2r, a_1 = 2r+1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+2} \bigodot P^4_{4s+2}$ is cordial.

**Case 7.** $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $L_{4r}001$ for $P_{4r+3}$. Take $\alpha_0$ to be $01_30S_{4s-4}0$ and $\alpha_1$ to be $0L_{4s-4}01_30$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = a_1 = 2r+1, y_0 = y_1 = 2s+1, b_0 = b_1 = 8s-1, y'_0 = y'_1 = 2s+1$ and $b'_0 = b'_1 = 8s-1$. It follows that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Hence, $P_{4r+3} \bigodot P^4_{4s+2}$ is cordial. □

**Lemma 3.5.** *If $n \equiv 3(mod\ 4)$, then $P_m \bigodot P^4_n$ is cordial for all $m \geq 1$.*

*Proof.* Suppose that $n = 4s+3$, where $s \geq 1$. The following cases will be checked.

**Case 1.**

Suppose that $m = 1$. Choose the labeling 0 for $P_1$ and the labeling $\alpha_0$= $1_2S_{4s}0$ for $P^4_{4s+3}$. Therefore $x_0 = 1, x_1 = 0, a_0 = a_1 = 0, y_0 = 2s+1, y_1 = 2s+2, b_0 = b_1 = 8s+1$ and obviously $y'_0 = y'_1 = b'_0 = b'_1 = 0$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_1 \bigodot P^4_{4s+3}$ is cordial.

**Case 2.**

Suppose that $m = 2$. Choose the labeling 01 for $P_2$. Take $\alpha_0$ to be $0_21L_{4s}$ and $\alpha_1$ to be $1_2S_{4s}0$. Therefore $x_0 = x_1 = 1, a_0 = 0, a_1 = 1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2$ and $b'_0 = b'_1 = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_2 \bigodot P^4_{4s+3}$ is cordial.

**Case 3.**

Suppose that $m = 3$. Choose the labeling 010 for $P_3$. Take $\alpha_0$ to be $0_21L_{4s}$, $\alpha_1$ to be $1_2S_{4s}0$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_3$) to be $1_2S_{4s}0$. Therefore $x_0 = 2, x_1 = 1, a_0 = 0, a_1 = 2, y_0 = 2s+2, y_1 = 2s+1, b_0 =$


[ DOI: 10.61186/ijmsi.18.2.107 ]

$b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2, b'_0 = b'_1 = 8s+1, y_0^{'*} = 2s+1, y_1^{'*} = 2s+2$ and$b_0^{'*} = b_1^{'*} = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_3 \bigodot P^4_{4s+3}$ is cordial.

**Case 4.** $m \equiv 0(mod\ 4)$.

Suppose that $m = 4r$, $r \geq 1$. Choose the labeling $M_{4r}$ for $P_{4r}$. Take $\alpha_0$ to be $0_2 1 L_{4s}$ and $\alpha_1$ to be $1_2 S_{4s} 0$. Therefore $x_0 = x_1 = 2r, a_0 = 0, a_1 = 4r-1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2$ and $b'_0 = b'_1 = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. As an example, Figure (4) illustrates $P_4 \bigodot P^4_7$. Hence, $P_{4r} \bigodot P^4_{4s+3}$ is cordial.

**Case 5.** $m \equiv 1(mod\ 4)$.

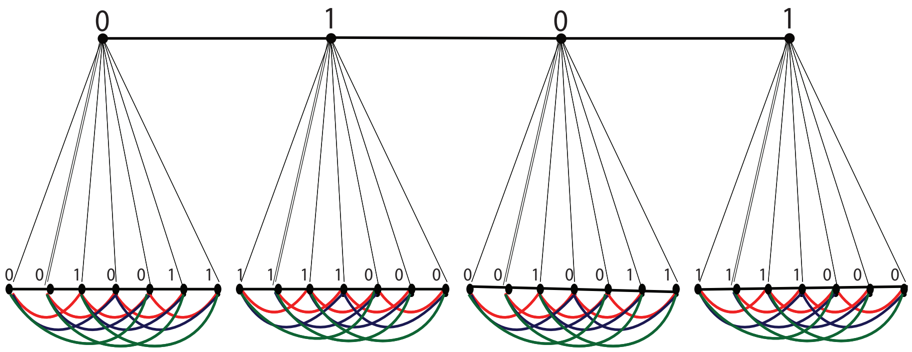


FIGURE 4

Suppose that $m = 4r+1$, $r \geq 1$. Choose the labeling $M_{4r}0$ for $P_{4r+1}$. Take $\alpha_0$ to be $0_2 1 L_{4s}$, $\alpha_1$ to be $1_2 S_{4s} 0$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_{4r+1}$) to be $1_2 S_{4s} 0$. Therefore $x_0 = 2r+1, x_1 = 2r, a_0 = 0, a_1 = 4r, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2, b'_0 = b'_1 = 8s+1, y_0^{'*} = 2s+1, y_1^{'*} = 2s+2$ and $b_0^{'*} = b_1^{'*} = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+1} \bigodot P^4_{4s+3}$ is cordial.

**Case 6.** $m \equiv 2(mod\ 4)$.

Suppose that $m = 4r+2$, $r \geq 1$. Choose the labeling $M_{4r+2}$ for $P_{4r+2}$. Take $\alpha_0$ to be $0_2 1 L_{4s}$ and $\alpha_1$ to be $1_2 S_{4s} 0$. Therefore $x_0 = x_1 = 2r+1, a_0 = 0, a_1 = 4r+1, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2$ and $b'_0 = b'_1 = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Hence, $P_{4r+2} \bigodot P^4_{4s+3}$ is cordial.

**Case 7.** $m \equiv 3(mod\ 4)$.

Suppose that $m = 4r+3$, $r \geq 1$. Choose the labeling $M_{4r+2}0$ for $P_{4r+3}$. Take $\alpha_0$ to be $0_2 1 L_{4s}$, $\alpha_1$ to be $1_2 S_{4s} 0$ and $\alpha{*}_0$ (associated to the last vertex labeled 0 in $P_{4r+3}$) to be $1_2 S_{4s} 0$. Therefore $x_0 = 2r+2, x_1 = 2r+1, a_0 = 0, a_1 = 4r+2, y_0 = 2s+2, y_1 = 2s+1, b_0 = b_1 = 8s+1, y'_0 = 2s+1, y'_1 = 2s+2, b'_0 =$

$b'_1 = 8s+1, {y'_0}^* = 2s+1, {y'_1}^* = 2s+2$ and ${b'_0}^* = {b'_1}^* = 8s+1$. It follows that $v_0 - v_1 = 0$ and $e_0 - e_1 = -1$. Hence, $P_{4r+3} \bigodot P^4_{4s+3}$ is cordial.
As a consequence of all lemmas mentioned above we conclude that the corona product between paths and fourth power of paths is cordial for all $m, n \geq 7$. $\square$

## Conclusion

In this paper we test the cordiality of the corona product between paths and fourth power of paths. We have shown that all $P_m \bigodot P^4_n$ are cordial for all integers $m \geq 1$ when $n \geq 7$, and for all integers $m > 1$ when $n = 3$.

## Acknowledgments

The authors wish to thank the referees, for the careful reading of the paper and for the helpful suggestions and comments.

## References

1. A. Rosa, *On Certain Valuations of the Vertices of a Graph*, Theory of Graphs (Internat Symposium, Rome, July 1966), Gordon and Breach, N. Y. Dunod Paris, 1967, 349-355.
2. B. D. Acharya, S. M. Hegde, Arithmetic Graphs, *J. Graph Theory*, **14**(3), (1990), 275-299.
3. I. Cahit, Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs, *Ars Combinatoria*, **23**, (1987), 201-207.
4. I. Cahit, On Cordial and 3-eqtitable Labeling of Graphs, *Utiliuies Math*, **37**, (1990).
5. I. Cahit, On Cordial Labeling of the Second Power of Paths with other Graphs, *ARS Combinatoria*, **97A**, (2010), 327-343.
6. J. A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics*, **DS6**, (2017).
7. S. Freeda, S. R. Chellathurai, $H_1$ and $H_2$ Cordial Labeling of Some Graphs, *Open J. Discrete Math*, **2**, (2012), 149-155.
8. S. I. Nada, A. T. Diab, A. Elrokh, D. E. Sabra, The Corona between Paths and Cycles, *Ars Combin*, accepted.
9. V. Yegnanarayanan, P. Vaidhyanathan, Some Interesting Applications of Graph Labellings, *Journal of Mathematical and Computational Science*, **2**(5), (2012), 1522-1531.